

Introduction to Impossible Probability, Statistics of Probability or Probabilistic Statistics. VOL 8

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Introduction to Impossible Probability,
Statistics of Probability or Probabilistic Statistics
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Translated into English in September 2025,
using ChatGPT, keeping the translation as faithful as possible to the original

Preliminary Words [2015 edition]

Introduction to Impossible Probability, Statistics of Probability or Probabilistic Statistics is one of those works in which a deep bond between mathematics and philosophy can be found.

It always asks about the ultimate purpose of science, wrapped in a veil of uncertainty and relativity.

There is always a halo of indeterminism, since in the end it is chance itself—the random variations of what we call reality, although we do not quite know what it is—that becomes the cause of causes.

From an interpretation not devoid of a certain nihilism, it offers a different vision of mathematics applied to statistics and probability.

The work is organized into a series of sections whose exposition unfolds in a continuous manner, each section and each paragraph being numbered in an ordered numerical sequence, in a way that facilitates reading.

Introduction to Impossible Probability first appeared in December 2011, in a limited edition of which a few copies still remain. It was later modified into a new eBook version that appeared in January 2013, whose first revision dates from January 2014. The version now presented is the latest update, based on contributions from January 2015.

To all readers, many thanks, and enjoy,
Rubén García Pedraza

Preliminary Words to the English Translation

The work presented here, *Introduction to Impossible Probability*, is essential for understanding the original meaning of the Global Artificial Intelligence. Its origins can be traced back to my writings in 2002 on Astro-ecology, the science that understands the cosmos as a single environment that must be studied from a unified, single science. In essence, this is what the Global Artificial Intelligence seeks to achieve. At the same time, in 2002 I developed the formulations of Impossible Probability, beginning in the early hours of September 11, 2001 with the *Impact of the Defect*, which was later expanded into the *Second Method* on October 16, 2002.

In essence, the entire original philosophy is based on the Cartesian idea of the necessity of a margin of doubt in any scientific proposition. This Cartesian margin of doubt would later, in the application of Karl Popper's falsificationism to inferential statistics, be understood as *critical reason*, the necessary and acceptable margin of error in any provisionally true proposition.

Impossible Probability maintains and deepens the Cartesian and Popperian rationalist tradition, enriching it with new philosophical contributions and some mathematical formulations. It rests on the understanding that the necessity of a margin of doubt in the existence of every provisional truth stems from the human limitation of being incapable of accessing the noumenon of things. It is this a priori limitation—the metaphysical limitation of the human condition—that requires humans to accept errors. Yet this does not mean that the noumenon cannot be known by other means. For this reason, in *Impossible Probability* we propose the construction of the Global Artificial Intelligence: a machine capable of making real the dream of a unified science, a single science that may lead us to the reason for all things. The creation of a global matrix, which may ultimately be resolved in a single equation, will provide the equation that solves the matrix, the very reason for everything.

Due to the special conditions in which I find myself—explained in other documents available on Internet Archive and Amazon—I am compelled to publish as quickly as I translate. For this reason, the English version of *Introduction to Impossible Probability* will be released in volumes as the translation progresses.

September 1, 2025, London, Leytonstone, 18:32
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“What did you do with the other guys

Did people brush their teeth

In the concentration camp”

Ruben Garcia Pedraza

Poem “*Mr Saddle*”

At Stratford Library Poetry Club

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17. Projection and Forecast

17.1. Prediction, Projection, and Forecast

17.1.1. To predict is, prior to the occurrence, to say what is going to happen; to predict is to anticipate.

17.1.2. Historically, prediction had mythical or mystical connotations; it was what prophets and **oracles** did, who in theory possessed clairvoyance [**At some point, the matrix, GAI, will be the post-human oracle**]. They formulated propositions of what could happen, verified in light of scarce objective knowledge, admitting a large margin of error due to the rudimentary state of their science. Predictions were formulated through the interpretation of natural phenomena—a mix of fantasy and partly objective knowledge accumulated since prehistory—and preserved by sorcerers, shamans, and priests, who accessed the mysteries through limited knowledge of the exact and natural sciences. They predicted the flooding of the Nile, an eclipse, or established the annual cycles of the seasons to elaborate calendars that organized sowing and harvesting, guided the economy, and established the first forms of social organization—early states, kingdoms, and empires—legitimized by *myths and legends that would later evolve into philosophy and science*.

17.1.3. Science has evolved toward forms of prediction grounded in empirical data, on which, based on observed trends, one can determine what may happen: scientific prediction.

17.1.4. The most elementary form of linear prediction is established through logic, such that if A then B. Logically, there is a correlation between A and B that, if sufficient, might imply that B is the effect of A. Once the causal relationship between A and B is established, where A is the cause of B, then B is the effect of A, and thus if A occurs, it is anticipated or predicted that B will occur. However, this relationship, apparently logical, is not absolutely reliable in practice.

17.1.5. The causal relationship between A and B can be presented empirically in two forms: in a subjectively direct form—that is, sensorially direct, for example, if the sun shines, the temperature rises—or in a stochastic form, in terms of probability, for example, of the number of times it rains, the probability that a rainbow will appear equals the ratio of the number of times the rainbow is observed to the total number of times it rains.

17.1.6. In the prediction “*if it rains, a rainbow may appear*,” we can observe the stochastic nature of knowledge. The possibility that a rainbow appears after rainfall is a function of probability. It is one possibility among many, but objectively it takes the form of a ratio. The truth is that if it rains, there is no absolute guarantee that a rainbow will always appear. What we can forecast is that, based on a probability ratio (the number of times the rainbow appears over a sufficient sample of rainy days), we can establish a mathematical ratio sufficient to predict the possibility that on any given rainy day a rainbow might appear.

17.1.7. The basis of empirical skepticism, founded by Hume, is that all truth propositions, including stochastic ones, are of a subjective, sensorial, *psychological* nature, which are taken as true based on custom, eventually becoming a sociological conventionalism. For idealism and rationalism, the relationship between psychology and science is closer and more complex than a mere synthesis of perceptual physiology and conventionalism or habit. The proposition “if A then B” is a product of reason, of human psychology. Human psychology, through logical reasoning, tends to

order factors, such that the relationship of A and B responds to a psychological structure, not necessarily to what is external. Reason itself tends to establish scientific relations between A and B under certain circumstances, which may, however, be nothing more than psychological relations of thought or reason that do not necessarily correspond to material reality. Within idealism and rationalism, there are two tendencies:

17.1.7.1. Classical idealism or rationalism—Plato or Descartes—where universal reason is itself truth, because what is external is mere appearance, falsehood. Thus, the maximum possible knowledge derives from reason itself: “know thyself,” since everything is deduced from one’s own thought, from reason. “*I think, therefore I am*”: only what reason declares to be true is truth.

17.1.7.2. For critical rationalism—Kant—rational knowledge is the product of reason rationalizing, critiquing what is external (phenomena), in accordance with transcendental psychological categories a priori, which produce absolute transcendental ideas for the knowledge of contingent phenomena. Transcendental ideas are the synthesis of a complete series of phenomena, whose deductions, beyond experience, can be the logical cause of antinomies when, in the face of a question, two opposing reasons, theses, or explanations arise, equally defensible by their respective series of premises.

17.1.7.3. The main difference between classical idealism or rationalism and critical rationalism is that while Plato or Descartes defend that reason by itself is universal, refuting appearances, Kant denies the possibility of knowledge of the noumenon, thereby creating the antinomy between the universal and the particular contingency. Rational knowledge is only possible through rational critique of phenomena. Rational knowledge is antinomic in nature, since the transcendent, universal idea is the synthesis of the series of accidents, from which it follows that knowledge must be provisional: Popper’s falsificationism. The contradiction between phenomenon and noumenon is resolved by Hegel in the dialectical synthesis, which admits the possibility of knowing the absolute idea, universal reason. Thus the essence, the noumenon, being-in-itself, reality-in-itself, can indeed be knowable.

17.1.7.4. *Impossible Probability*, as a philosophical synthesis, only provisionally accepts empirical hypotheses as true when accepted in rational critique, being in all cases partial knowledge, not absolute knowledge of reality itself. It makes hypotheses politically true within the margin of error accepted by scientific policy: between political truth (for science itself) and reality itself (of the universe). This margin of skepticism or doubt, in practice, becomes the genesis of future scientific revolutions, when incoherencies that cause crises in science are overcome by newer, deeper, and superior hypotheses that refute previous ones, tending toward a stationary state of permanent scientific revolution.

17.1.8. Idealist and rationalist philosophy, from Plato to Descartes, was strongly influenced by linear geometry of Euclidean origin. From the 19th century onwards, nonlinear geometric models began to emerge, and nonlinear logic began to rise in contemporary times with quantum physics, chaos theories, and complexity.

17.1.9. Since the basis of nonlinear relations are linear relations, the foundation of dialectics is logic. One cannot understand nonlinear logic without first understanding linear logic. Linear logic is the foundation of the theory of prediction, which in its most complex form may even be nonlinear prediction.

17.1.10. In *Impossible Probability*, two types of linear predictions will be formulated: a critical prediction of future behavior in relation to the ideal (projection) [this is the reason why projects are located on the second stage in the Decisional System in GAI, the main function of a project is to achieve all AI Ideals], and an empirical prediction of the future in relation to the observed empirical trend between two measurements (forecast) [second stage Modelling System in GAI].

17.1.11. Thus, in *Impossible Probability* there will be two forms of prediction: a projective prediction based on ideals, and a real prediction based on empirical trends themselves [the synthesis of both is the Global Project System, our GPS, this is the real global plan].

17.1.12. Projection or critical prediction to the ideal will predict what the behavior of future occurrences should be in order to sufficiently tend toward the ideal, within the margin of logical or moral incoherence accepted between the ideal and real reality.

17.1.13. Forecast, or empirical or real prediction, will predict what the real or empirical behavior of an occurrence should be, consistent with its observed trend, within the margin of logical uncertainty or causal relativism that scientific policy accepts.

17.1.14. In prediction—whether projections or forecasts—the need for error margins is explained by the fact that, being a priori estimations based on linear logic, while real reality is nonlinear, the insignificant may become decisive, the impossible may become possible. If there is no absolute certainty of anything, absolute affirmations are illogical; therefore, predictions can only be provisional, historical, or relative, within a margin of uncertainty.

17.1.15. If chance determines everything, and everything determines chance, there are no sufficient reasons to justify that an observed trend must persist over time. Thus, any linear projection based on an observed trend can only be accepted within a margin of error or incoherence, between the forecast and what scientific policy is truly willing to accept.

17.1.16. If any accident, however insignificant it may seem, can determine the future, any ideal projection must be a **project of the future** within the margin of incoherence we are willing to accept between our political ideals and what may really happen.

17.1.17. Any linear logic will truly be logical only insofar as it accounts for an error margin regarding the nonlinear reality it seeks to explain or understand.

17.1.18. Since reality is the product of accidents by chance, nothing guarantees that the distribution of matter will remain linear forever, hence the need for margins of uncertainty. By chance, there is a tendency for the top of demographic pyramids to show a higher rate of women than men. But this is a random, natural, chaotic tendency—except under eugenic policy or genetic engineering. By chance, each year a certain number of meteorites impact our planet, a distribution of accidents that follows a random distribution. At most, we can improve detection of giant meteorites whose trajectories approach Earth, and politically, within a margin of error, create planetary shields to prevent them from colliding.

17.1.19. Science does not make absolute predictions; *prophecy* does not exist. One type of prediction is linear prediction under the hypothesis that a trend will remain unchanged—an assumption that must be questioned, because reality is the result of accidents of the universe. In space-time, nothing guarantees that observed or ideal patterns will be repeated or reproduced. Every linear projection or forecast depends entirely on the hypothesis of continuity or maintenance of a trend, assuming chance, and under the supposition that no unfavorable events occur—something that cannot be guaranteed scientifically at one hundred percent. The only thing science can do, when faced with chance, is to intervene scientifically, through scientific praxis, reducing margins of error to increase reliability.

17.2. Concept of Projection and Forecast

17.2.1. Projected Probability

17.2.1.1. The *project*, in *Impossible Probability*, will be that predictive act upon an empirical *model* deduced from a prior reality—first measurement—so that, through linear sequencing, the possible model of future reality can be anticipated on the basis of an ideal [in 2011 I did not make any distinction between projects and models, actually in the sixth phase of GAI projects and models are placed in a shared space, at some point dialectically, models and projects and projects are models].

17.2.1.2. *If in a growth study the ideal objective for a given subject or option is that its empirical probability should reach the Maximum Possible Empirical Probability—probability one—whether due to an experimental variable in an experimental study*, or in the study of natural or social trends, where the trend initially observed shows a tendency to grow, possible results could be projected for later measurements from the initial or prior measurement, estimating the *Projected Probability*: the sum of the initial or prior empirical probability (first measurement) plus the Reliable Possible Variation—Critical or Empirical—to the ideal, whether in experimental study or in the evolution of natural or social trends. To this result error margins may be added, within which the empirical probability obtained in the future measurement is expected to fall, in order to determine whether the empirical

probability in the future measurement is within the expected error margin of the *Projected Probability*.

17.2.1.3. In studies of decrease, once an initial or prior downward trend is observed, the initial empirical probability is reduced by the Reliable Possible Variation—Critical or Empirical—toward the ideal of the Minimum Possible Empirical Probability (probability zero), to determine the *Projected Probability*. To this estimate, error margins can be added, within which the empirical probability of the future measurement is expected to fall, whether in an experimental study or in observed trends.

17.2.1.4. In equality of opportunity, whether experimental or an evolving trend, once the trend toward equality of opportunity is determined, the *Projected Probability* is estimated: initial or prior empirical probability plus Reliable Possible Variation—Critical or Empirical—if *the aim is to reduce negative bias*, or initial or prior empirical probability minus Reliable Possible Variation—Critical or Empirical—if *the aim is to reduce positive bias*, to determine the *Projected Probability*, with error margins finally added.

17.2.1.5. The error margin will always be the addition or subtraction of a critical probability to the result of the addition or subtraction of the initial or prior empirical probability (first measurement) to the Reliable Possible Variation. The narrower the error margin, the less error is accepted and the more demanding the study. The wider the error margin, the less demanding it is.

17.2.1.6. Since any project, based on a linear projection of the observed trend, will depend on the determination by chance of the trend in future events, the logic of linear projections in *Impossible Probability* will be the following: from a first measurement, projections are made regarding what should be the *Projected Probability* of a subject or option in the future, based on a trend observed in the first measurement and in relation to the ideal trend. For example, in a growth trend the ideal trend will be toward the Maximum Possible Empirical Probability (probability one); in a decrease trend the ideal trend will be toward the Minimum Possible Empirical Probability (probability zero); and in equality of opportunity, the ideal trend will be toward the inversion of N. The empirical trend of the first measurement toward the ideal trend is studied through the Reliable Possible Variation—Critical or Empirical—so that it is projected that in future

measurements the subject or option should reach a certain margin of Reliable Possible Variation—Critical or Empirical—toward the ideal. That Reliable Possible Variation, added (in growth) or subtracted (in decrease) to the initial measurement, determines the *Projected Probability*, which may be Critical or Empirical, since it represents what the subject or option is projected to tend toward materially in order to reach the possible ideal, according to its capacity of Reliable Possible Variation. Afterwards, the *Projected Probability* is contrasted with the empirical probability that the subject or option reaches in the future measurement.

17.2.1.7. Since every project is an idealization of reality, what must be projected is always the way the ideal should be realized; therefore, *every project, to begin with, must be reliable.*

17.2.1.8. In *Projected Probability* through Reliable Possible Variation—whether in equality or bias, positive or negative—in principle all values within the limits of the error margin will be accepted. In the study of reliable positive bias, in *Projected Probability*, scientific policy may accept any future measurement equal to or greater than the lower limit or, if stricter, only those equal to or greater than the upper limit. In the study of reliable negative bias, in *Projected Probability*, scientific policy may accept any future measurement equal to or less than the upper limit, or, if stricter, only those equal to or less than the lower limit.

17.2.1.9. There will be two types of *Projected Probability*: Critical Projected Probability and Empirical Projected Probability, depending on whether the *Projected Probability* is estimated based on the Critical or Empirical Reliable Possible Variation.

17.2.1.10. Critical Projected Probability will be projected so that the necessary variation is multiplied by a critical probability of reliability, percentage “Xa” of reliability out of one hundred, and the error margin is calculated over a critical probability of error, percentage “Xb” of error out of one hundred. Thus, the reliability margin by which the necessary variation is multiplied will be designated “ $p(xc/a)$,” and the error margin will be designated “ $\pm p(xc/b)$.”

17.2.1.11. Every Critical Projected Probability will therefore be established on a reliability percentage “Xa” and an error percentage “Xb.”

17.2.2. Forecasted Probability

17.2.2.1. Every projection is the linear projection to the ideal based on the first measurement: given a first measurement, what should be the trend in terms of Reliable Possible Variation toward the ideal.

17.2.2.2. To calculate *Projected Probability*, it is only necessary to make a first measurement and identify the ideal trend, then multiply the absolute value of the difference between the ideal and the empirical probability of the first measurement by reliability, empirical or critical, thereby calculating the Reliable Possible Variation. Thus, *Projected Probability* is the addition or subtraction, to the empirical probability of the first measurement, of the Reliable Possible Variation, and to the result, an error margin is added or subtracted. Finally, future measurements are contrasted against the error margins. This is, in essence, the logic of *Projected Probability*. *Projected Probability* prepares a predictive project on the Reliable Possible Variation—Critical or Empirical—toward the ideal trend.

17.2.2.3. The logic of *Forecasted Probability* changes radically: if two measurements have been made, then the real variation between the two measurements equals the difference in empirical probability that a subject or option may have between the two measurements.

17.2.2.4. Within *Impossible Probability*, three different types of variations exist:

- **17.2.2.4.1.** *Necessary variation*, the difference between two factors, whether normal or relative Bias Level, or the difference between an empirical probability and the Maximum or Minimum Possible Empirical Probability. In absolute form, subtracted from the greater factor of the differential, it is the decrement needed for the greater to equal the lesser; or, added to the lesser factor in absolute value, it is the increment the lesser must reach to equal the greater.
- **17.2.2.4.2.** There are two types of *Possible Variation*: by Chance or Reliable, depending on whether the necessary variation is multiplied by an error or reliability term. Thus, Reliable Possible Variation equals the product of the necessary variation by empirical or critical reliability.

- **17.2.2.4.3.** *Real variation* is the variation that has actually occurred between two different measurements. In growth, real variation equals the difference of the second measurement minus the first measurement—that is, the difference of the empirical probability of the second measurement minus that of the first measurement. In decrease, real variation equals the difference of the first measurement minus the second measurement—that is, the difference of the empirical probability of the first measurement minus that of the second measurement. At the sample level, real variation in the study of bias equals the difference of the Mean or Standard Deviation (DMoS) of the second measurement minus that of the first measurement; in the study of equality of opportunity, the Mean or Standard Deviation of the first measurement minus that of the second.

17.2.2.5. Necessary variation is, for example, the normal or relative Bias Level, intrameasurement. Reliable Possible Variation is, for example, the product of the normal or relative Bias Level, intrameasurement, by empirical or critical reliability, or the difference between empirical probability and Maximum or Minimum Possible Empirical Probability by empirical or critical reliability. Reliable Possible Variation functions to indicate to what extent a variation from a first measurement to a future measurement is reliable. Real variation is the difference between two different real measurements, to know the true variation that actually occurs between them.

17.2.2.6. If real variation is the variation that actually occurs from the first to the second measurement, then the first measurement plus, in growth, the real variation equals the value of the second measurement. Necessarily, if to the value of the second measurement in growth one adds the real variation, one should really forecast the value of the future measurement, provided the rate of change between the second and third measurement remains constant, as it did between the first and second. Thus, in growth studies, *Forecasted Probability*, within linear logic, equals the sum of the empirical probability of the second measurement plus the real variation, with the real variation being equal to the difference between the empirical probability of the second and that of the first.

17.2.2.7. In decrease studies, *Forecasted Probability* equals the subtraction of the real variation from the empirical probability of the second measurement, with the real variation being equal to the difference between the empirical probability of the first measurement and that of the second.

17.2.2.8. The *Forecasted Mean or Standard Deviation* (DMoS) in bias studies will equal the sum of the Mean or Standard Deviation of the second measurement plus the real variation, with the real variation equal to the difference between the Mean or Standard Deviation of the second and that of the first, being a forecast consistent with linear logic.

17.2.2.9. The *Forecasted Mean or Standard Deviation* in equality of opportunity will equal the Mean or Standard Deviation of the second measurement minus the real variation, with the real variation equal to the difference between the Mean or Standard Deviation of the first and that of the second.

17.2.2.10. If error margins were added to the projected prediction, error margins may likewise be added to the forecasted prediction. Any result in the future measurement that falls within the forecasted error margin is accepted. In growth studies, more specifically, there is the option to accept any result equal to or greater than the lower limit, or, if stricter, only those results equal to or greater than the upper limit. In decrease studies, more specifically, there is the option to accept any future measurement equal to or less than the upper limit of the error margin, or, if stricter, only those equal to or less than the lower limit of the error margin.

17.2.3. Differences between Projected Probability and Forecasted Probability

17.2.3.1. *Projected Probability* is the prediction made from a first measurement regarding the *Reliable Possible Variation* toward an ideal trend, whereas *Forecasted Probability* is the prediction made on the *real variation*. *Reliable Possible Variation* is the product of the necessary variation and reliability (empirical or critical); real variation equals the difference between two measurements.

17.2.3.2. *Projected Probability* is a plan for what the future measurement should be so that the *Reliable Possible Variation* toward the ideal becomes real, whereas

Forecasted Probability is a forecast—based on the observed real variation of growth or decrease—of what the actual future measurement should be.

17.2.3.3. The main difference between *Projected Probability* and *Forecasted Probability* is that *Projected Probability* predicts—within the accepted error margin—what the possible individual trend toward the ideal should be, while *Forecasted Probability* predicts—within the accepted error margin—what the individual trend should actually be in reality.

17.2.3.4. *Projected Probability* predicts an ideal future; *Forecasted Probability* predicts a real future—logically, within a linear logic.

17.2.3.5. Dialectically, everything real is ideal, and everything ideal is real, within the error margin one is willing to accept; beyond the error there is only metaphysics—within the error margin there are only provisional truths of reality-in-itself. Outside science, any proposition lacks scientific value; within science, any proposition is only provisional. Within the partial limitation of the human being, the relation between possible ideal and possible reality is a function that depends on time. In a limited, partial, sample-based reality, only what is possible in the sample will be possible. What is impossible in a given sample of space-time may be impossible for that particular sample of space-time, but **what is impossible for one sample of space-time may even be inevitable in a different sample of space-time**. What for a given sample may perhaps not be, may be absolutely inevitable for another different sample. The condition of being necessary, in practice, will depend on its logical coherence with reality-in-itself; in reality-in-itself only what is coherent for that reality-in-itself happens, although, for a set of different realities-in-themselves, anything different in itself may happen. What is incoherent for one particular reality may be absolutely coherent for another distinct reality-in-itself. All dialectic is, in essence, the dialectic of being and not-being.

17.3. Projected Probability for a Second Measurement

17.3.1. Concept of Projected Probability in a Second Measurement

17.3.1.1. In a normal dispersion model where empirical dispersion can range from zero to the maximum possible dispersion, if it is a study of equality of opportunity, the ideal is that—given a trend toward equality of opportunity—the empirical probability of the subject or option tends to be the inversion of N .

17.3.1.2. In a normal dispersion model where empirical dispersion can range from zero to the maximum possible dispersion, if the study's ideal is that a given subject or option reaches the greatest possible positive bias, the ideal is that—given an empirical probability tending to increase positive bias—the empirical probability tends toward the Maximum Possible Empirical Probability, empirical probability equal to one.

17.3.1.3. In the study of negative bias, given an empirical probability one wishes to reduce to the minimum possible, the ideal will be the Minimum Possible Empirical Probability, probability zero. The empirical probability will increase negative bias as it decreases as much as possible. Within studies of negative bias there are different options: either the ideal is a sample of zeros—then the ideal would be dispersion, individual or sample, equal to the absolute value of the inversion of N ; or the aim is that there be N minus one subjects or options tending to negative bias and only a single subject or option tending to maximum positive bias; or within omega models that there be a magnitude equal to N minus omega subjects or options tending to increase negative bias. In any case, omega models for intermeasurement studies will be addressed later.

17.3.1.4. Given a study, in general, the study's ideal will determine what the ideal trend of the empirical probabilities should be.

17.3.1.4.1. In a study of equality of opportunity, the ideal is that empirical probabilities tend to be the inversion of N ; therefore, those empirical probabilities with positive bias ideally should tend to reduce positive bias, and the empirical probabilities with negative bias should ideally tend to reduce negative bias.

17.3.1.4.2. In a study of positive bias, the ideal is that the empirical probability that is to increase positive bias to the maximum tends to be empirical probability equal to one; therefore, that positive-bias empirical probability will tend to increase to become the Maximum Possible Empirical Probability.

17.3.1.4.3. In a study of negative bias, in any kind of model, the ideal is that the empirical probability that is to increase negative bias tends to be empirical probability equal to zero; the empirical probability will tend to be the Minimum Possible Empirical Probability.

17.3.1.5. Saying that, depending on the study's trend, an empirical probability must grow or decrease means that this growth or decrease in the trend will depend on the *necessary variation*—how much the probability must grow or decrease to equal that other ideal factor.

17.3.1.6. If in a study of equality of opportunity the normal Bias Level must decrease, then the normal Bias Level is the necessary variation that must occur between the first and second measurement so that the empirical probability becomes equal to the inversion of N in the second measurement.

17.3.1.7. If in a study of positive bias, the difference of one—the Maximum Possible Empirical Probability—minus the empirical probability is the amount by which the empirical probability must grow to equal the Maximum Possible Empirical Probability, then that is the necessary variation for the empirical probability to equal the Maximum Possible Empirical Probability in the second measurement.

17.3.1.8. If in a study of negative bias, the difference of the empirical probability minus zero equals the empirical probability itself, then the empirical probability itself is the necessary variation by which the empirical probability must decrease to be zero in the second measurement.

17.3.1.9. The *necessary variation* will then be, in the projection to the ideal, that amount of variation required for an empirical probability to become equal to the ideal; the ideal trend is the trend of every subject or option to be morally what it ought to be.

17.3.1.10. However, *not everything necessary is always possible*. Therefore, to determine what part of the necessary variation is *reliably* to be projected, the necessary variation is multiplied by reliability—empirical or critical—to determine the *Reliable Possible Variation*, Critical or Empirical.

17.3.1.11. Reliable Possible Variation is that part of the necessary variation that is required for a change in behavior to be reliable, and it is the product of the necessary variation and a reliability term—for example, necessary variation times critical probability as a reliability margin (percentage X of reliability out of one hundred), equal to *Critical Reliable Possible Variation*. In empirically reliable projection studies, multiply necessary variation by empirical reliability to obtain *Empirical Reliable Possible Variation*, with Empirical Reliability computed in the standard way. **First Mode of Empirical Reliability, “ $\neg\alpha$,” in studies of equality of opportunity: one minus the quotient of dividing the Mean or Standard Deviation (normal) by the Maximum Theoretical Possible Mean or Standard Deviation. Second Mode of Empirical Reliability, “ β ,” if the object of study is maximum dispersion: equal to the Mean or Standard Deviation** (normal) divided by the Maximum Theoretical Possible Mean or Standard Deviation. Examples using DM (Mean Deviation), although explained in more detail in section sixteen [I will keep DM and MSoD, but these symbols are Spanish denominations].

17.3.1.11.1. First Mode of Empirical Reliability, $\neg\alpha$, using Mean Deviation (DM), in equality of opportunity:

$$\neg\alpha = 1 - \{DM : \{[(1 - 1/N) \cdot 2] : N\}\}$$

DM= Mean Deviation

17.3.1.11.2. Second Mode of Empirical Reliability, β , using Mean Deviation, when the object of study is maximum dispersion:

$$\beta = \{DM : \{[(1 - 1/N) \cdot 2] : N\}\}$$

DM=Mean Deviation

17.3.1.12. In tests of *Projected Probability*, every value within or equal to the error margins will be accepted, with *Projected Probability* equal to the empirical probability of the first measurement plus (in growth) or minus (in decrease) the *Reliable Possible Variation*—Critical or Empirical—Individual; and to the result, error margins are added, within which **moral judgments (statistical decisions)** will be issued according to rational critique and scientific policy. Only in *Projected Probability* studies of positive bias is there the option to accept any second measurement equal to or greater than the lower limit or, if stricter, equal to or greater than the upper limit; or, in the study of negative bias, to accept any second measurement equal to or less than the upper limit or, if stricter, equal to or less than the lower limit, in accordance with what scientific policy establishes.

17.3.1.13. Generically, all possible models of *Projected Probability*, all equally, will be symbolized with the symbol “ $p(x_i)'$ ”; that is, the symbol of empirical probability with a prime is the symbol used to denote *Projected Probability*. In fact, in the first phase of *Impossible Probability*—especially in 2003—I often referred to the projected value simply as “empirical probability prime.”

$p(x_i)'$ = Projected Probability

17.3.1.14. *Projected Probability* for the second measurement would then be symbolized as “ $p(x_{i/2nd})'$ ”:

$p(x_{i/2nd})'$ = Projected Probability for the second measurement

17.3.1.15. To make the concepts absolutely clear for the upcoming equations—fundamental in the Second Method—recall that the normal Bias Level is the differential relation of empirical probability minus the inversion of N . When the empirical probability is greater than the inversion of N , there is said to be positive bias. When the empirical probability is less than the inversion of N , there is said to be negative bias.

17.3.1.15.1. Those *Projected Probability* equations that are exclusively for reducing positive bias must be applied only to those empirical probabilities greater than the inversion of N:

$$p(x_i) > 1/N \rightarrow [(p(x_i) > 1/N) - 1/N] = \text{positive bias}$$

17.3.1.15.2. Those *Projected Probability* equations that are exclusively for reducing negative bias must be applied only to those empirical probabilities lower than the inversion of N.

$$p(x_i) < 1/N \rightarrow [(p(x_i) < 1/N) - 1/N] = \text{negative bias}$$

17.3.1.16. Calculating the error margin

17.3.1.16.1. Both *Projected Probability* and, later, *Forecasted Probability* will have associated error margins within which future measurements are accepted, depending on whether the future measurements fall within the error margin; or—in the study of reliable positive bias—whether the future measurement is equal to or greater than the lower limit, or only equal to or greater than the upper limit; and—in the study of reliable negative bias—whether it is equal to or less than the upper limit, or only equal to or less than the lower limit. The error margin in prediction is a function determined by a *critical probability* which, as an error margin, is added to or subtracted from the prediction; and the critical probability in *Projected Probability* will be symbolized as “ $\pm p(x_c/b)$,” as the error margin by which the critical probability is added or subtracted.

$\pm p(xc/b) = \text{error margin}$

$\text{Prediction} + p(xc/b) = \text{upper limit of the error margin}$

$\text{Prediction} - p(xc/b) = \text{lower limit of the error margin}$

17.3.1.16.2. The value of the prediction itself may be the value of the *Projected Probability* or the *Forecasted Probability*. The value of the prediction plus the error margin equals the upper limit; the value of the prediction minus the error margin equals the lower limit.

The value of the *Projected Probability* itself equals, in growth, the empirical probability of the first measurement plus the *Reliable Possible Variation*; in decrease, the empirical probability of the first measurement minus the *Reliable Possible Variation*.

The value of the *Forecasted Probability* itself equals, in growth, the sum of the empirical probability of the second measurement plus the real variation; in decrease, the empirical probability of the second measurement minus the real variation.

The error margin is a critical probability which, when added to the value of the *Projected or Forecasted Probability*, gives the upper limit of the error margin; and the value of the *Projected or Forecasted Probability* minus the critical probability gives the lower limit of the error margin.

17.3.1.16.3. In the study of positive bias, the ideal is that the *Projected or Forecasted Probability* tends to the *Maximum Possible Empirical Probability*, i.e., one. In probability theory—at least in statistical probability—if probability is the ratio between the part and the whole, the part cannot exceed the whole; at most it can equal the whole, but never exceed it. It follows that a probability greater than one would be entirely irrational. If a *Forecasted Probability* were greater than one, one would simply say that the *Maximum Possible Empirical Probability* is forecast.

17.3.1.16.4. In the study of negative bias, the ideal is that the *Projected* or *Forecasted Probability* tends to the *Minimum Possible Empirical Probability*, zero. In fact, it would be entirely irrational for the empirical probability of a positive event to be a negative-signed probability, because a positive event can only have a positive-signed empirical probability—or, if the positive event is “that something does not occur,” the probability would be zero that the event occurs; but in any case, never—and absolutely never—can any event have a negative-signed probability. **If a *Forecasted Empirical Probability* were less than zero (negative), one would simply say that the *Minimum Possible Empirical Probability* [zero] is forecast.**

17.3.1.16.5. In the study of equality of opportunity, the ideal will be to tend toward the inversion of N.

17.3.1.16.6. The error margin is, in essence, a margin of moral flexibility [cognition flexibility] within which to accept a sufficient tendency toward the ideal.

17.3.1.16.7. The error margin in the study of a trend toward maximum positive bias is a margin of moral flexibility within which to accept that something may tend sufficiently toward the *Maximum Possible Empirical Probability*.

17.3.1.16.8. The error margin in the study of negative bias is a margin of moral flexibility within which to accept that something may tend sufficiently toward the *Minimum Possible Empirical Probability*.

17.3.1.16.9. The error margin in equality of opportunity is a margin of moral flexibility within which to accept that something may tend sufficiently toward the inversion of N.

17.3.1.16.10. If the error margin is essentially a margin of moral flexibility in linear prediction, then the error margin must be set within the range of difference between the ideal and the *Projected* or *Forecasted Probability* itself.

17.3.1.16.11. The error margin (or moral flexibility) in *Projected Probability* for the study of maximum positive bias equals the product of the error percentage X_b (out of one hundred) by the difference between one (the maximum ideal possible in the study of positive bias) and the *Projected Probability* itself; that is, the product of $X_b/100$ and the difference of one minus the *Projected Probability* itself equals the error margin in

Projected Probability for positive bias.

Critical probability in positive bias studies=

$\pm p(xc/b) = \{ 1 - (p(xi/1st) + VP) \} \cdot (Xb : 100)$ = error margin in the study of positive bias

$p(xi/1st)$ = empirical probability in the first measurement

VP = *Reliable Possible Variation* [$VP=Variacion Posible$, in Spanish]

Xb = error percentage

17.3.1.16.12. The error margin (or moral flexibility) in *Projected Probability* for the study of negative bias will be directly the same *Projected Probability* itself—inasmuch as it is the factor that must decrease to reach the ideal, zero—times the error percentage X_b out of one hundred.

Critical probability in negative bias studies=

$\pm p(x_c/b) = (p(x_i/1st) - VP) \cdot (X_b : 100) = \text{error margin in the study of negative bias}$

$p(x_i/1st)$ = empirical probability in the first measurement

VP = *Reliable Possible Variation*

X_b = error percentage

17.3.1.16.13. The error margin (or moral flexibility) in *Projected Probability*, in equality of opportunity, is the product of the error percentage X_b (out of one hundred) and the absolute value of the difference between the inversion of N and the *Projected Probability* itself—whether first measurement plus or minus *Reliable Possible Variation* to reduce negative or positive bias.

Critical probability in studies of equal opportunities=

$\pm p(x_c/b) = \{ 1/N - (p(x_i/1st) \pm VP) \} \cdot (X_b : 100) = \text{error margin in equality of opportunity}$

$p(x_i/1st)$ = empirical probability in the first measurement

VP = *Reliable Possible Variation*

X_b = error percentage

17.3.2. Projected Probability, Critical or Empirical

17.3.2.1. Critical Projected Probability

17.3.2.1.1. Critical Projected Probability in the study of equality of opportunity

17.3.2.1.1.1. *Critical Projected Probability* in the study of equality of opportunity, to reduce positive bias:

$$p(x_i/2nd)' = \{ p(x_i) - [(p(x_i) - 1/N) \cdot p(x_c/a)] \} \pm p(x_c/b)$$

$$p(x_c/a) = X_a : 100$$

X_a = reliability percentage

$$p(x_i/2nd)' = \{ p(x_i) - [(p(x_i) - 1/N) \cdot p(x_c/a)] \} + p(x_c/b) = \text{upper limit of the error margin}$$

$$p(x_i/2nd)' = \{ p(x_i) - [(p(x_i) - 1/N) \cdot p(x_c/a)] \} - p(x_c/b) = \text{lower limit of the error margin}$$

$$\pm p(x_c/b) = / \{ 1/N - \{ p(x_i) - [(p(x_i) - 1/N) \cdot p(x_c/a)] \} \} / \cdot (X_b : 100)$$

X_b = error percentage

17.3.2.1.1.2. Critical Projected Probability in the study of equality of opportunity, **to reduce negative bias:**

$$p(x_i/2nd)' = \{ p(x_i) + [(1/N - p(x_i)) \cdot p(x_c/a)] \} \pm p(x_c/b)$$

$$p(x_c/a) = X_a : 100$$

X_a = reliability percentage

$$p(x_i/2nd)' = \{ p(x_i) + [(1/N - p(x_i)) \cdot p(x_c/a)] \} + p(x_c/b) = \text{upper limit of the error margin}$$

$$p(x_i/2nd)' = \{ p(x_i) + [(1/N - p(x_i)) \cdot p(x_c/a)] \} - p(x_c/b) = \text{lower limit of the error margin}$$

$$\pm p(x_c/b) = \{ 1/N - \{ p(x_i) + [(1/N - p(x_i)) \cdot p(x_c/a)] \} \} \cdot (X_b : 100)$$

X_b = error percentage

17.3.2.1.2. Critical Projected Probability, in the study of bias

17.3.2.1.2.1. Critical Projected Probability, in the study of bias, to increase the empirical probability to the *Maximum Possible Empirical Probability*:

$$p(x_i/2nd)' = \{ p(x_i) + [(1 - p(x_i)) \cdot p(x_c/a)] \} \pm p(x_c/b)$$

$$p(x_c/a) = X_a : 100$$

X_a = reliability percentage

$$p(x_i/2nd)' = \{ p(x_i) + [(1 - p(x_i)) \cdot p(x_c/a)] \} + p(x_c/b) = \text{upper limit of the error margin}$$

$$p(x_i/2nd)' = \{ p(x_i) + [(1 - p(x_i)) \cdot p(x_c/a)] \} - p(x_c/b) = \text{lower limit of the error margin}$$

$$\pm p(x_c/b) = \{ 1 - \{ p(x_i) + [(1 - p(x_i)) \cdot p(x_c/a)] \} \} \cdot (X_b : 100)$$

X_b = error percentage

17.3.2.1.2.2. Critical Projected Probability, in the study of bias, **to reduce the empirical probability to the Minimum Possible Empirical Probability:**

$$p(x_i/2nd)' = \{ p(x_i) - [p(x_i) \cdot p(x_c/a)] \} \pm p(x_c/b)$$

$$p(x_c/a) = X_a : 100$$

X_a = reliability percentage

$$p(x_i/2nd)' = \{ p(x_i) - [p(x_i) \cdot p(x_c/a)] \} + p(x_c/b) = \text{upper limit of the error margin}$$

$$p(x_i/2nd)' = \{ p(x_i) - [p(x_i) \cdot p(x_c/a)] \} - p(x_c/b) = \text{lower limit of the error margin}$$

$$\pm p(x_c/b) = \{ p(x_i) - [p(x_i) \cdot p(x_c/a)] \} \cdot (X_b : 100)$$

X_b = error percentage

17.3.2.2. Empirical Projected Probability, in the study of equality of opportunity.

First mode of Empirical Reliability

17.3.2.2.1. Empirical Projected Probability, in the study of equality of opportunity, **to reduce positive bias. First mode of Empirical Reliability:**

$$p(x_i/2nd)' = \{ p(x_i) - [(p(x_i) - 1/N) \cdot \alpha] \} \pm p(x_c/b)$$

$$p(x_i/2nd)' = \{ p(x_i) - [(p(x_i) - 1/N) \cdot \alpha] \} + p(x_c/b) = \text{upper limit of the error margin}$$

$$p(x_i/2nd)' = \{ p(x_i) - [(p(x_i) - 1/N) \cdot \alpha] \} - p(x_c/b) = \text{lower limit of the error margin}$$

$$\pm p(x_c/b) = / \{ 1/N - \{ p(x_i) - [(p(x_i) - 1/N) \cdot \alpha] \} \} / \cdot (X_b : 100)$$

X_b = error percentage

17.3.2.2.2. Empirical Projected Probability, in the study of equality of opportunity, **to reduce negative bias. First mode of Empirical Reliability:**

$$p(x_i/2nd)' = \{ p(x_i) + [(1/N - p(x_i)) \cdot \neg\acute{\alpha}] \} \pm p(x_c/b)$$

$$p(x_i/2nd)' = \{ p(x_i) + [(1/N - p(x_i)) \cdot \neg\acute{\alpha}] \} + p(x_c/b) = \text{upper limit of the error margin}$$

$$p(x_i/2nd)' = \{ p(x_i) + [(1/N - p(x_i)) \cdot \neg\acute{\alpha}] \} - p(x_c/b) = \text{lower limit of the error margin}$$

$$\pm p(x_c/b) = \{ 1/N - \{ p(x_i) + [(1/N - p(x_i)) \cdot \neg\acute{\alpha}] \} \} \cdot (X_b : 100)$$

X_b = error percentage

17.3.2.3. Empirical Projected Probability, in the study of bias. Second mode of Empirical Reliability

17.3.2.3.1. *Empirical Projected Probability*, in the study of bias, to increase the empirical probability to the *Maximum Possible Empirical Probability*. Second mode of Empirical Reliability:

$$p(x_i/2nd)' = \{ p(x_i) + [(1 - p(x_i)) \cdot \beta] \} \pm p(x_c/b)$$

$$p(x_i/2nd)' = \{ p(x_i) + [(1 - p(x_i)) \cdot \beta] \} + p(x_c/b) = \text{upper limit of the error margin}$$

$$p(x_i/2nd)' = \{ p(x_i) + [(1 - p(x_i)) \cdot \beta] \} - p(x_c/b) = \text{lower limit of the error margin}$$

$$\pm p(x_c/b) = \{ 1 - \{ p(x_i) + [(1 - p(x_i)) \cdot \beta] \} \} \cdot (X_b : 100)$$

X_b = error percentage

17.3.2.3.2. Empirical Projected Probability, in the study of bias, **to reduce the empirical probability to the *Minimum Possible Empirical Probability*. Second mode of Empirical Reliability:**

$$p(x_i/2nd)' = \{ p(x_i) - [p(x_i) \cdot \beta] \} \pm p(x_c/b)$$

$$p(x_i/2nd)' = \{ p(x_i) - [p(x_i) \cdot \beta] \} + p(x_c/b) = \text{upper limit of the error margin}$$

$$p(x_i/2nd)' = \{ p(x_i) - [p(x_i) \cdot \beta] \} - p(x_c/b) = \text{lower limit of the error margin}$$

$$\pm p(x_c/b) = \{ p(x_i) - [p(x_i) \cdot \beta] \} \cdot (X_b : 100)$$

X_b = error percentage

17.3.3. Tests of Critical or Empirical Projected Probability, with respect to the second measurement

17.3.3.1. Tests of Critical or Empirical Projected Probability, with respect to the second measurement, when the two measurements have the same N

17.3.3.1.1. *A priori*, when contrasting the empirical results of a second measurement against the projected values, every value within or equal to the error margins is accepted.

17.3.3.1.2. In the study of positive bias, one may accept any second measurement equal to or greater than the lower limit of the error margin, or—if stricter—only those equal to or greater than the upper limit.

17.3.3.1.3. In the study of negative bias, one may accept any second measurement equal to or less than the upper limit, or—if stricter—only those equal to or less than the lower limit.

17.3.3.2. Tests of Critical or Empirical Projected Probability, with respect to the second measurement, when the second measurement has a different N

17.3.3.2.1. In the tests of Critical or Empirical Projected Probability, “ $p(x_i/2nd)'$ ” with respect to the empirical probability measured in the second measurement, “ $p(x_i/2nd)$,” if it turns out that between the first measurement (from which the projections were calculated) and the second measurement, the samples of each measurement have different N, then the contrasts—of projections from one measurement with N different from the N of the second measurement—must be tests carried out on the *effect of N*.

$p(x_i/2nd)'$ = Projected Probability, Critical or Empirical, for the second measurement

$p(x_i/2nd)$ = empirical probability of the second measurement

17.3.3.2.2. There are two ways to calculate the effect of N in the tests of Projected Probability—Empirical or Critical—either by multiplying the effect of N by the Projected Probability:

Effect of N on Projected Probability = $p(x_i/2nd-m_i)'$ · ($N_{mi} : N_m$)

$p(x_i/2nd-m_i)'$ = Projected Probability designated “ m_i ” [projected based on first measurement]

N_{mi} = N magnitude of the first measurement

N_m = N magnitude of the second measurement

17.3.3.2.3. Or by calculating the product of the effect of N by the empirical probability of the second measurement:

Effect of N on the empirical probability of the second measurement =

$$p(x_i/2^{nd}-m_i) \cdot (N_{mi} : N_m)$$

$p(x_i/2^{nd}-m_i)$ = empirical probability of the second measurement designated “mi”

N_m = N magnitude of the first measurement

N_{mi} = N magnitude of the second measurement

17.3.3.2.4. In any case, it will always be absolutely necessary—if tests are made between projections from a measurement with N different from the N of the second measurement—to adhere to the rational criterion that the most effective approach is to simplify the equations.

17.3.3.2.5. If the effect of N is applied to the Projected Probability, then the “ N_{mi} ” measurement is the N of the first measurement from which the forecast has been made, and the “ N_m ” measurement is the second measurement. The product of the Projected Probability by the effect of N will imply the product of the effect of N both by the upper limit of the error margin and by the lower limit of the error margin, in order to be contrasted with the empirical probability of the second measurement—provided that the second measurement differs in N magnitude from the first measurement.

17.3.3.2.6. However, if the effect of N is applied to the empirical probability of the second measurement, then “ N_{mi} ” is the N of the second measurement, and “ N_m ” is the N of the first measurement from which the prediction was made. Thus, by multiplying the effect of N by the empirical probability of the second measurement, the

result of that product can be compared directly with the limits of the *Projected Probability's* error margin. This second option—multiplying the effect of N by the empirical probability of the second measurement—when the N of the second measurement differs from the N of the first, will be the most suitable for simplifying the calculation of the *Projected Probability*.

17.3.3.2.7. Otherwise, the contrast criteria will be the same as in other situations, once the effect of N has been calculated for the empirical probability of the second measurement when the second measurement differs in N. In principle, any second measurement—adjusted by the effect of N—should be accepted if it lies within the error margin. In studies of positive bias, scientific policy may accept those measurements—adjusted by the effect of N—that are equal to or greater than the lower limit of the error margin, or, if stricter, only those equal to or greater than the upper limit of the error margin. In studies of negative bias, accept all measurements—adjusted by the effect of N—equal to or less than the upper limit of the error margin, or, if stricter, only those equal to or less than the lower limit of the error margin.

17.3.4. The necessity of error margins in projections

17.3.4.1. When making a projection about the future, it is always advisable to allow error margins, since by accepting an error margin one sets a range within which it is rational to accept an empirical hypothesis about the future.

17.3.4.2. If *Projected Probability* is calculated without error margins, then the probability that the results of the second measurement will match the projected estimates—without accepting any error margin—is too low, because the decisions allow no latitude for error.

17.3.4.3. By giving latitude for error in decisions, it is allowed that, even if in a second measurement the empirical probability is not exactly equal to the computation of the empirical probability of the first measurement plus or minus (depending on growth or decrease) the *Reliable Possible Variation*, the fact that the second measurement's empirical probability is not, in decrease, exactly equal to the first measurement's empirical probability minus the *Reliable Possible Variation*, or, in growth, not exactly equal to the first measurement's empirical probability plus the *Reliable Possible*

Variation, is not required by the model, since some flexibility is granted. If the empirical probability of the second measurement tends sufficiently to match the projection—hypothetically, within the error margin—the second measurement will be accepted. The error margin allows room to accept trends that tend to be close to the hypothetical projected values for the future measurement, and to accept any future measurement value that, while not exactly equal to the projected value, tends to be equal—within the error margin—to the projected value; hypothetically, any trend of that future measurement that lies within the error margin and tends sufficiently toward the projected value will tend to be accepted as sufficient.

17.3.4.4. By accepting error margins, it is not required, in growth, that the second measurement equal the sum of the first measurement plus the *Reliable Possible Variation*, because there is an error margin within which the trend of the second measurement may fluctuate and still be accepted. The error margin becomes a margin of moral flexibility for accepting the empirical hypothesis.

17.3.4.5. Likewise, by allowing error margins, in decrease it is not required that the second measurement be exactly equal to the first measurement minus the *Reliable Possible Variation*, since there is a margin of moral flexibility for accepting the empirical hypothesis and an error margin within which the trend of the second measurement is accepted, as long as it tends sufficiently to approach the value hypothetically projected.

17.3.4.6. By creating an error margin within which a statistical decision about what may happen in the future is accepted, one acknowledges the human limitation of being unable to predict the future with absolute reliability. Hence an error margin is created as a margin of flexibility, because in science one cannot make absolutely reliable predictions when human nature is itself limited. If science is incomplete or provisional—and therefore coherent only provisionally—it is irrational to believe that the future is absolutely predictable. At most, science can make statistical predictions: predictions within what is possible given partial knowledge of reality; scientific predictions far removed from the classical concept of prophecy. One can only anticipate or be proactive regarding *the trend*, and only based on the study of the trend. In the Second Method all possible knowledge is knowledge of the trend. The

fundamental object of statistics is the trend. The analytical method of *Impossible Probability* is the syllogism of the trend.

17.3.4.7. In science there are no prophets, oracles, or chosen ones, because only what happens exists, and everything that happens usually happens because, from all that is possible—finite or infinite—the universe of possible events ultimately yields *some* occurrence, whether by chance or with sufficient reliability. Every event by definition is statistical. Statistics is the history of chance—or the chance of history—in any case, history and chance: what happens in space-time.

17.3.4.8. In fact, under chance, nothing is necessary; everything is accidental or contingent. The only necessity, given a set of possible options, alternatives, subjects, or objects, is that something occurs—even if the event is a direct score or frequency of zero—whenever it is possible. The sample of zeros is also a positive fact in itself. When it is said that a variation is *necessary*, it will be necessary according to the objectives or ideals of the scientific investigation, which requires that particular necessary variation—and no other—in order to achieve the objectives and ideals of scientific policy.

17.3.5. Projected Probability in “nth” measurements

17.3.5.1. Concept of nth *Reliable Possible Variation*

17.3.5.1.1. Up to now, *Projected Probability*—Critical or Empirical—has been calculated to predict what the *Projected Probability* should be in a second measurement, based on the data obtained in a first measurement.

17.3.5.1.2. However, in a study one may be able to make more than one subsequent measurement, both in descriptive and inferential models (and within inferential models, experimental ones), in order to study the evolution of the behavior of matter according to our projects, and to produce predictive estimates of behavior across different measurements, following linear logic, according to the premises of the straight line.

17.3.5.1.3. Within a dialectical logical philosophy, it is clear that real processes do not always follow linear schemes. In fact, nonlinear logic is the basis of dialectics, insofar

as—for dialectics—not all real processes follow a linear model; if the insignificant can be decisive, then dialectical logic (which implies the determination of everything by chance, and of chance by everything—the order of factors does not alter the product) must necessarily follow processes that are not always linear. Within dialectical logic, understanding that not all processes are linear and that nonlinear mathematics offers enormous fields of inquiry for the development of explanatory models—especially since quantum physics, mathematical models of chaos and complexity, and non-Euclidean geometries—one must also understand that the basis of all nonlinear logic is linear logic. If Euclidean geometry is not understood, it is difficult to understand non-Euclidean geometries; for this reason, if the basis of nonlinear logic is linear mathematics, linear projections along the straight line are necessary—while remaining aware that, in reality, processes do not always follow linear schemes. Hence it is necessary to introduce, into all linear logic, error margins, in order to account for the error between the hypothetically linear projected or forecast event and the real (nonlinear) movement of matter.

17.3.5.1.4. In an investigation, a series of measurements can be scheduled; the total number of measurements to be carried out will be denoted by the symbol “M,” which will represent the total number of measurements.

M = total number of measurements

17.3.5.1.5. Each particular measurement will be denoted “me,” a symbol that will represent each of the *nth* measurements [me= *mediciones emesimas*, in Spanish]. The term “nth measurement” refers to each of the particular measurements that will make up the total set “M” of measurements scheduled in an investigation. The symbol “me” for *nth* measurements—each of the particular measurements forming “M”—is interpreted as follows: “me” is composed of the letter “m,” meaning measurement, and the letter “e,” the ordinal number (*nth*) of the particular measurement to which “me” corresponds; thus each *nth* measurement “me” must be ordered according to the chronological time of its measurement number.

$M = \{ m_1, m_2, m_3 \dots m_e \}$

m_e = any particular measurement, nth measurement [*mediciones emesimas*]

m_1 = first measurement

m_2 = second measurement

m_3 = third measurement

17.3.5.1.6. The empirical probability of the nth measurement is symbolized by “ $p(x_i/m_e)$ ”.

$p(x_i/m_e)$ = empirical probability of the nth measurement

17.3.5.1.7. In a research project where different nth measurements are scheduled, over a total of “M” different measurements, estimating statistical predictions for only the second measurement would be insufficient; it is necessary to make predictions for each of the different nth measurements that make up “M.”

17.3.5.1.8. For example, if “M” different measurements are scheduled so that equality of opportunity is reached in a model, then, based on the first measurement, predictive estimates must be made regarding what results should be projected or forecast for each of the subsequent scheduled nth measurements: project results for the second measurement; project or forecast results for the third measurement; project or forecast results for the fourth measurement; and so on up to the last nth measurement.

17.3.5.1.9. The projected probability for each of the n th measurements will be called the *nth Projected Probability* and will be symbolized by the n th empirical probability prime “ $p(x_i/m_e)'$ ”.

$p(x_i/m_e)' =$ Projected Probability of the n th measurement

17.3.5.1.10. If, for a study of equality of opportunity, based on a first measurement, projected predictive estimates must be made for the subsequent n th measurements, then the product of (each n th measurement index minus one), in order to exclude the first measurement, by the quotient of the normal *Reliable Possible Variation* over (M minus one), will be the *normal nth Reliable Possible Variation*. Thus, in the study of equality of opportunity, in order to reduce positive bias, (empirical probability minus the normal *nth Reliable Possible Variation*), or, in order to reduce negative bias, (empirical probability plus the normal *nth Reliable Possible Variation*), will equal the projected estimate predicted for each of the subsequent n th measurements.

17.3.5.1.11. The *nth Projected Probability*, in a bias study, will be equal to either: to **increase positive bias**, the sum of the empirical probability plus the *nth Reliable Possible Variation*; or, to **increase negative bias**, the empirical probability minus the *nth Reliable Possible Variation*.

$p(x_i/m_e)' = p(x_i) + VPF_n$ (increase positive bias)

$p(x_i/m_e)' = p(x_i) - VPF_n$ (increase negative bias)

where $VPF_n =$ *nth Reliable Possible Variation*. [**Variacion Posible Fiable, Spanish**]

17.3.5.1.12. The *nth Reliable Possible Variation* will always equal the *Reliable Possible Variation* divided by the number of M measurements minus one, multiplied by the difference between the *nth* measurement to be projected and one.

$$\text{nth Reliable Possible Variation} = [\text{VPF} : (M - 1)] \cdot (\text{me} - 1)$$

$\text{VPF} = \text{Reliable Possible Variation}$

M = total number of measurements

me = *nth* measurement

17.3.5.1.13. The reason why the quotient *Reliable Possible Variation* over (M minus one) is multiplied by (nth measurement minus one) is very simple. If the data on which subsequent statistical predictions are made are obtained from the first measurement, then the difference “nth measurement minus one” for the first measurement equals zero; the product of zero by the *Reliable Possible Variation* over (M minus one) would be zero. Therefore, if the *Projected Probability* equals the empirical probability of the first measurement plus (in growth) or minus (in decrease) the *nth Reliable Possible Variation*, then—logically—if the *nth Reliable Possible Variation* is zero for the first measurement, necessarily the first measurement’s empirical probability plus or minus zero equals the first measurement’s empirical probability. Necessarily, if the projections are made from the empirical probability of the first measurement, the logical thing is that the estimates for the first measurement do not change for that first measurement; they are, in any case, predictive estimates for *future* measurements. For this reason, the result of the quotient—*Reliable Possible Variation* over (M minus one)—is multiplied by (nth measurement minus one), so that the scientific estimates do not vary for the first measurement, and the future estimates are for the future *nth* measurements. Since the estimates are made starting from the first measurement, one

multiplies the *nth Reliable Possible Variation* by (nth measurement minus one); logically, the *Possible Variation* must be divided by (M minus one), since of all the measurements M there is one that is not estimated in the projections—the first measurement. Hence the *nth Reliable Possible Variation* equals the *Reliable Possible Variation* divided by (M minus one).

17.3.5.1.14. The tests (contrasts) of *Projected Probabilities* for nth measurements are the same as for the other forms of *Projected Probabilities*.

17.3.5.1.15. Normally, any empirical probability of an nth measurement that lies within the error margin of the *Projected Probability* will be accepted. In a study of positive bias, scientific policy may accept any empirical probability of an nth measurement that is equal to or greater than the lower limit of the error margin, or—if stricter—only those empirical probabilities of the nth measurement that are equal to or greater than the upper limit of the error margin. In a study of negative bias, accept any empirical probability of an nth measurement that is equal to or less than the upper limit of the error margin, or—if stricter—only those empirical probabilities of nth measurements that are equal to or less than the lower limit of the error margin.

17.3.5.1.16. If all the nth measurements that make up the set of M measurements have the same sample N, then the empirical probabilities of each nth measurement can be compared directly against the error margin of the *Projected Probabilities*.

17.3.5.1.17. If any of the nth measurements that make up the set of M measurements have an N different from the sample N of the first measurement (from which the data were linearly derived for the subsequent nth measurements), then, in order to contrast the empirical probabilities of nth measurements with N different from that of the first measurement, the *effect of N* must be applied to the empirical probability of those nth measurements that have a different N. The *effect of N* equals the N magnitude of the nth measurement divided by the N magnitude of the first measurement, and the effect of N is multiplied by the resulting empirical probabilities of that nth measurement that has a different N, so that the empirical probabilities of the nth measurement with different N can be contrasted against the *Projected Probability* for that nth

measurement.

Effect of N in nth measurement = $p(x_i/m_i - m_i) \cdot (N_{mi} : N_m)$

$p(x_i/m_i - m_i)$ = empirical probability of nth measurement designated “ m_i ”

(N_{mi} = N of the nth measurement “ m_i ”; N_m = N of the first measurement.)

17.3.5.1.18. Insofar as the preceding examples made it clear that in *Critical Projected Probability* the necessary variation is multiplied by a critical probability of reliability “ $p(x_c/a)$,” based on a reliability percentage “ X_a ” out of one hundred, with an error margin denoted “ $p(x_c/b)$,” calculated on an error percentage “ X_b ” out of one hundred, by the difference between the ideal and the projection, in the following examples we will proceed directly to present only the calculation of the error margin—since the reliability margin is always the same (reliability percentage “ X_a ” out of one hundred times the necessary variation). Only the method for calculating the error margin changes, depending on the object of study, as it is the product of the error percentage “ X_b ” out of one hundred and the difference between the maximal or ideal trend and the projection itself.

17.3.5.2. Projected Probability, Critical or Empirical, in nth measurements

17.3.5.2.1. Critical Projected Probability in nth measurements

17.3.5.2.1.1. Critical Projected Probability in the study of equality of opportunity, in nth measurements

17.3.5.2.1.1.1. Critical Projected Probability in the study of equality of opportunity, to reduce positive bias, in nth measurements:

$$p(x_i/me)' = \{ p(x_i) - \{ \{ [(p(x_i) - 1/N) \cdot p(x_c/a)] : (M - 1) \} \cdot (me - 1) \} \} \pm p(x_c/b)$$

$$\pm p(x_c/b) = / \{ 1/N - \{ p(x_i) - \{ \{ [(p(x_i) - 1/N) \cdot p(x_c/a)] : (M - 1) \} \cdot (me - 1) \} \} \} / \cdot (X_b : 100)$$

X_b = error percentage

17.3.5.2.1.1.2. Critical Projected Probability in the study of equality of opportunity, to reduce negative bias, in nth measurements:

$$p(x_i/me)' = \{ p(x_i) + \{ \{ [(1/N - p(x_i)) \cdot p(x_c/a)] : (M - 1) \} \cdot (me - 1) \} \} \pm p(x_c/b)$$

$$\pm p(x_c/b) = \{ 1/N - \{ p(x_i) + \{ \{ [(1/N - p(x_i)) \cdot p(x_c/a)] : (M - 1) \} \cdot (me - 1) \} \} \} \cdot (X_b : 100)$$

X_b = error percentage

17.3.5.2.1.2. Critical Projected Probability in the study of bias, in nth measurements

17.3.5.2.1.2.1. Critical Projected Probability in the study of bias, to increase the empirical probability to the *Maximum Possible Empirical Probability*, in nth measurements:

$$p(x_i/me)' = \{ p(x_i) + \{ \{ [(1 - p(x_i)) \cdot p(x_c/a)] : (M - 1) \} \cdot (me - 1) \} \} \pm p(x_c/b)$$

$$\pm p(x_c/b) = \{ 1 - \{ p(x_i) + \{ \{ [(1 - p(x_i)) \cdot p(x_c/a)] : (M - 1) \} \cdot (me - 1) \} \} \} \cdot (X_b : 100)$$

X_b = error percentage

17.3.5.2.1.2.2. Critical Projected Probability in the study of bias, to reduce the empirical probability to the *Minimum Possible Empirical Probability*, in nth measurements:

$$p(x_i/me)' = \{ p(x_i) - \{ \{ [p(x_i) \cdot p(x_c/a)] : (M - 1) \} \cdot (me - 1) \} \} \pm p(x_c/b)$$

$$\pm p(x_c/b) = \{ p(x_i) - \{ \{ [p(x_i) \cdot p(x_c/a)] : (M - 1) \} \cdot (me - 1) \} \} \cdot (X_b : 100)$$

X_b = error percentage

17.3.5.2.2. Empirical Projected Probability. First Mode of Empirical Reliability, in nth measurements, in the study of equality of opportunity

17.3.5.2.2.1. Empirical Projected Probability, First Mode of Empirical Reliability, in nth measurements, to reduce positive bias:

$$p(x_i/me)' = \{ p(x_i) - \{ \{ [(p(x_i) - 1/N) \cdot \alpha] : (M - 1) \} \cdot (me - 1) \} \} \pm p(x_c/b)$$

$$\pm p(x_c/b) = / \{ 1/N - \{ p(x_i) - \{ \{ [(p(x_i) - 1/N) \cdot \alpha] : (M - 1) \} \cdot (me - 1) \} \} \} / \cdot (X_b : 100)$$

X_b = error percentage

17.3.5.2.2.2. Empirical Projected Probability, First Mode of Empirical Reliability, in nth measurements, to reduce negative bias:

$$p(x_i/me)' = \{ p(x_i) + \{ \{ [(1/N - p(x_i)) \cdot \alpha] : (M - 1) \} \cdot (me - 1) \} \} \pm p(x_c/b)$$

$$\pm p(x_c/b) = \{ 1/N - \{ p(x_i) + \{ \{ [(1/N - p(x_i)) \cdot \alpha] : (M - 1) \} \cdot (me - 1) \} \} \} \cdot (X_b : 100)$$

X_b = error percentage

17.3.5.2.3. Empirical Projected Probability. Second Mode of Empirical Reliability, in nth measurements, when the object of study is bias

17.3.5.2.3.1. Empirical Projected Probability, in the study of bias, to increase the empirical probability to the *Maximum Possible Empirical Probability*. Second Mode of Empirical Reliability, in nth measurements:

$$p(x_i/me)' = \{ p(x_i) + \{ \{ [(1 - p(x_i)) \cdot \beta] : (M - 1) \} \cdot (me - 1) \} \} \pm p(x_c/b)$$

$$\pm p(x_c/b) = \{ 1 - \{ p(x_i) + \{ \{ [(1 - p(x_i)) \cdot \beta] : (M - 1) \} \cdot (me - 1) \} \} \} \cdot (X_b : 100)$$

X_b = error percentage

17.3.5.2.3.2. Empirical Projected Probability, in a bias study, to reduce the empirical probability to the *Minimum Possible Empirical Probability*. **Second Mode of Empirical Reliability**, in nth measurements:

$$p(x_i/me)' = \{ p(x_i) - \{ \{ [p(x_i) \cdot \beta] : (M - 1) \} \cdot (me - 1) \} \} \pm p(x_c/b)$$

$$\pm p(x_c/b) = \{ p(x_i) - \{ \{ [p(x_i) \cdot \beta] : (M - 1) \} \cdot (me - 1) \} \} \cdot (X_b : 100)$$

X_b = error percentage

17.4. Projected Dispersion

17.4.1. Concept of Projected Dispersion

17.4.1.1. Given a first measurement, and within the object of study—the type of trend the study focuses on (equality of opportunity or bias, whether positive or negative)—there is sufficient information to estimate *Projected Probabilities* according to that first measurement and the type of trend under study (equality of opportunity or bias, positive or negative).

17.4.1.2. In a study, to demonstrate that a hypothesis of future trend is true, given a first measurement, *Projected Probabilities* are established for a second or nth measurement. Depending on whether the empirical probabilities of the second or nth measurements accord with the projections established from the first measurement, the empirical hypothesis will be rationally accepted or rejected.

17.4.1.3. With a first measurement in hand, one can already make future projective estimates from that measurement's data to rationally validate the empirical hypothesis about the future trend. Up to now, the process for calculating *Projected Probabilities* has been explained; in the same way that *Projected Probabilities* can be calculated, *Projected Dispersion* can also be calculated.

17.4.1.4. The *Impossible Probability* method is the syllogism of the trend. The very concept of the method also properly defines the object of study: what is really studied are trends, and empirically the sample's trend is the sample's empirical dispersion—normally the Mean Deviation or Standard Deviation.

17.4.1.5. In studies of equality of opportunity, the object will normally be to *reduce* the sample's empirical dispersion, so that subjects or options tend to behave around the inversion of N.

17.4.1.6. In standard bias studies (positive or negative), the object will be to *increase* the sample's empirical dispersion to the maximum possible, so that biased behavior of subjects or options tends to be as different as possible from the inversion of N.

17.4.1.7. Since what is done continuously is to study the sample's trend (in order to reduce or increase sample bias), any projective study of reality aimed at studying

the trend can, quite simply—in addition to discerning what each subject’s or option’s *Projected Probability* should be to understand the individual reality—also project what the *sample’s empirical dispersion* should be, i.e., the *Projected Dispersion*.

17.4.1.8. *Projected Dispersion* in a bias study equals the sample’s empirical dispersion from the first measurement **plus** the product of reliability (critical or empirical) and the difference between the *maximum possible empirical dispersion* and the first measurement’s empirical dispersion. In equality of opportunity—since the sample’s empirical dispersion itself is the factor to be decreased—the *Projected Dispersion* equals the *difference* of the sample’s empirical dispersion **minus** the product of that same sample empirical dispersion and reliability (empirical or critical).

17.4.1.9. The following models are developed using *normal Mean Deviation* [DM], since in the Second Method there is a predominance of Mean Deviation over Standard Deviation. The following equations are written with **Mean Deviation, preserving the original sense of the data (they are not squared nor square-rooted)**. To adapt this to the First Method, one would simply replace *Mean Deviation* with *Standard Deviation* where indicated, and replace *Maximum Theoretical Possible Mean Deviation* with *Maximum Theoretical Possible Standard Deviation* where necessary.

17.4.1.10. The symbol for *projected normal Mean Deviation* will be Mean Deviation prime, “DM ´”.

DM ´ = Projected normal Mean Deviation

DM(2nd) ´ = Projected normal Mean Deviation for the second measurement

DM(me) ´ = Projected normal Mean Deviation for the *n*th measurement

17.4.1.11. If adapted to the First Method (projecting the normal Standard Deviation), the symbol for *projected normal Standard Deviation* should be Standard Deviation prime “S’”.

S’ = Projected normal Standard Deviation

S(2nd)’ = Projected normal Standard Deviation for the second measurement

S(me)’ = Projected normal Standard Deviation for the nth measurement

[While DM preserve data, S is most commonly used, especially in Gauss models, the foundational house of modern statistics]

17.4.1.12. The error margin “ $\pm p(xc/b)$ ” for calculating the lower and upper limits in each model of *Projected Empirical Dispersion* depends on the ideal.

17.4.1.12.1. The error margin for *Projected Mean Deviation* in bias studies equals the product of the error percentage X_b (out of one hundred) and the difference between the *Maximum Theoretical Possible Mean Deviation* and the *Projected Mean Deviation itself*, where the *Projected Mean Deviation itself* equals the sum of the first measurement's Mean Deviation plus the *Reliable Possible Variation* (Empirical or Critical), *sample-level*.

Error margin in Projected Mean Deviation in a bias study:

$$\pm p(x_c/b) = \{ \{ [(1 - 1/N) \cdot 2] : N \} - (DM + VPF) \} \cdot (X_b : 100)$$

$$(DM + VPF) + p(x_c/b) = \text{upper limit of the error margin}$$

$$(DM + VPF) - p(x_c/b) = \text{lower limit of the error margin}$$

$VPF = \text{Reliable Possible Variation}$

$DM = \text{Mean Deviation of the first measurement}$

$X_b = \text{error percentage}$

17.4.1.12.2. The error margin for *Projected Mean Deviation* in equality of opportunity equals the error percentage X_b (out of one hundred) times the *Projected Mean Deviation itself*, where the *Projected Mean Deviation itself* is the difference between the first measurement's Mean Deviation and the *Reliable Possible Variation* (Empirical or Critical), *sample-level*.

Error margin in Projected Mean Deviation in equality of opportunity:

$$\pm p(xc/b) = (\mathbf{DM - VPF}) \cdot (X_b : 100)$$

$$(DM - VP) + p(xc/b) = \text{upper limit of the error margin}$$

$$(DM - VP) - p(xc/b) = \text{lower limit of the error margin}$$

$VPF = \text{Reliable Possible Variation}$

$DM = \text{Mean Deviation of the first measurement}$

$X_b = \text{error percentage}$

17.4.2. Projected normal Mean Deviation for a second measurement

17.4.2.1. *Critically Projected Mean Deviation*, in a bias study, for a second measurement:

$$DM(2nd)' = \{ DM + \{ [\{ (1 - 1/N) \cdot 2 \} : N \} - DM] \cdot p(xc/a) \} \pm p(xc/b)$$

DM= Mean Deviation first measurement

$$\pm p(xc/b) = \{ [\{ (1 - 1/N) \cdot 2 \} : N \} - \{ DM + \{ [\{ (1 - 1/N) \cdot 2 \} : N \} - DM] \cdot p(xc/a) \} \} \cdot (Xb : 100)$$

Xb = error percentage

17.4.2.2. *Critically Projected Mean Deviation*, in equality of opportunity, for a second measurement:

$$DM(2nd)' = \{ DM - [DM \cdot p(xc/a)] \} \pm p(xc/b)$$

DM= Mean Deviation first measurement

$$\pm p(xc/b) = \{ DM - [DM \cdot p(xc/a)] \} \cdot (Xb : 100)$$

Xb = error percentage

17.4.2.3. Empirically Projected Mean Deviation, in equality of opportunity, for a second measurement. **First Mode of Empirical Reliability:**

$$DM(2nd)' = \{ DM - [DM \cdot \alpha] \} \pm p(xcb)$$

DM= Mean Deviation first measurement

$$\pm p(xcb) = \{ DM - [DM \cdot \alpha] \} \cdot (Xb : 100)$$

Xb = error percentage

17.4.2.4. Empirically Projected Mean Deviation, in a bias study, for a second measurement. **Second Mode of Empirical Reliability**

$$DM(2nd)' = \{ DM + \{ [\{ (1 - 1/N) \cdot 2 : N \} - DM] \cdot \beta \} \} \pm p(xc/b)$$

$$\pm p(xc/b) = \{ [\{ (1 - 1/N) \cdot 2 : N \} - \{ DM + \{ [\{ (1 - 1/N) \cdot 2 : N \} - DM] \cdot \beta \} \}] \cdot (Xb : 100) \}$$

Xb = error percentage

17.4.3. Projected Mean Deviation for nth measurements

17.4.3.1. *Critically Projected Mean Deviation*, in a bias study, for nth measurements

$$DM(me)' = \{ DM + \{ \{ [(1 - 1/N) \cdot 2] : N \} - DM \} \cdot p(xc/a) \} : (M - 1) \} \cdot (me - 1) \} \pm p(xc/b)$$

$$\pm p(xc/b) = \{ [(1 - 1/N) \cdot 2] : N \} -$$

$$\{ DM + \{ \{ [(1 - 1/N) \cdot 2] : N \} - DM \} \cdot p(xc/a) \} : (M - 1) \} \cdot (me - 1) \} \cdot (Xb : 100)$$

Xb = error percentage

17.4.3.2. *Critically Projected Mean Deviation*, in equality of opportunity, for nth measurements

$$DM(me)' = \{ DM - \{ [DM \cdot p(xc/a)] : (M - 1) \} \cdot (me - 1) \} \pm p(xc/b)$$

$$\pm p(xc/b) = \{ DM - \{ [DM \cdot p(xc/a)] : (M - 1) \} \cdot (me - 1) \} \cdot (Xb : 100)$$

Xb = error percentage

17.4.3.3. Empirically Projected Mean Deviation, in equality of opportunity, for nth measurements. **First Mode of Empirical Reliability**

$$DM(me)' = \{ DM - \{ \{ [DM \cdot \neg \acute{\alpha}] : (M - 1) \} \cdot (me - 1) \} \} \pm p(xc/b)$$

$$\pm p(xc/b) = \{ DM - \{ \{ [DM \cdot \neg \acute{\alpha}] : (M - 1) \} \cdot (me - 1) \} \} \cdot (Xb : 100)$$

Xb = error percentage

17.4.3.4. Empirically Projected Mean Deviation, in a bias study, for nth measurements. **Second Mode of Empirical Reliability**

$$DM(me)' = \{ DM + \{ \{ \{ [(1 - 1/N) \cdot 2] : N \} - DM \} \cdot \beta \} : (M - 1) \} \cdot (me - 1) \} \} \pm p(xc/b)$$

$$\pm p(xc/b) = \{ \{ [(1 - 1/N) \cdot 2] : N \} -$$

$$\{ DM + \{ \{ \{ [(1 - 1/N) \cdot 2] : N \} - DM \} \cdot \beta \} : (M - 1) \} \cdot (me - 1) \} \} \cdot (Xb : 100)$$

Xb = error percentage

17.4.4. Interpretation of the contrast between Projected Mean Deviation and the Mean Deviation of a second or nth measurement

17.4.4.1. Any Mean Deviation of a second or nth measurement—in short, any Mean Deviation of a future measurement—that lies within the error-margin limits, or equals those limits, is accepted *a priori*.

17.4.4.2. In a bias study, one may accept any dispersion equal to or greater than the *lower* limit of the error margin, or—if stricter—only dispersion equal to or greater than the *upper* limit.

17.4.4.3. In a study of equality of opportunity, one may accept any dispersion equal to or less than the *upper* limit of the error margin, or—if stricter—only dispersion equal to or less than the *lower* limit.

17.4.4.4. If the Projected Dispersion is computed on a first-measurement sample with N different from the N of the second or nth measurement, then multiply the *effect of N* by the future measurement's Mean or Standard Deviation (second or nth) so it can be contrasted with the Projected Mean or Standard Deviation—whenever the future measurement's Mean or Standard Deviation has an N different from that of the first measurement. Thus, the future measurement is designated “mi” and the first measurement “m”; the effect of N is the quotient of the future measurement's N magnitude “Nmi” over the first measurement's N magnitude “Nm”. The effect of N times the future Mean Deviation measured, if it differs in N magnitude, equals the product of that future Mean Deviation by the quotient of its N magnitude over the first-measurement

N

magnitude.

N of the future measurement = Nmi

N of the first measurement = Nm

Effect of N = Nmi : Nm

Effect of N times the future measurement's Mean or Standard Deviation (DMoS) =

DMoS-mi · (Nmi : Nm)

17.5. Forecasted Empirical Probability

17.5.1. *Projected Empirical Probability* equals, in growth, the empirical probability of the first measurement **plus** *Reliable Possible Variation*; in decrease, the empirical probability of the first measurement **minus** *Reliable Possible Variation*. The *Reliable Possible Variation* used in *Projected Probability* is (a) the normal Bias Level, or (b) the difference between the Maximum or Minimum Possible Empirical Probability and the empirical probability, multiplied by a normal critical or empirical reliability margin, in order to make reliable normal projections. Finally, an error margin is added to the result. In any case, these are normal projections about what should be a sufficiently reliable trend within a normal dispersion model between zero and the maximum. *Omega* dispersion models, for samples with N greater than two and with an ideal number of subjects or options greater than one and less than N (between two and N minus one), for intermeasurement tests, will be studied in section twenty.

17.5.2. In *Impossible Probability* there are three variations: the *necessary variation* between two factors, the *Possible Variation*, and the *real variation*. The *necessary variation* is the variation the greater factor needs to *decrease* to equal the lesser factor

in a future measurement, or the variation the lesser factor needs to *increase* to equal the greater factor in a future measurement. The *Reliable Possible Variation* is the product of the necessary variation and empirical or critical reliability, to determine what part of the necessary variation would be reliable in a future measurement. Since the necessary variation estimates what variation would be required between two measurements to reach a given trend in a future measurement, and the *Reliable Possible Variation* is that variation by which a factor should vary to be reliable, the difference obtained between a first measurement and the empirical results finally obtained when making the second measurement will equal the *real variation*. *Real variation* is the variation that actually occurs in the difference between two measurements taken on the same factor.

17.5.3. Given two measurements of a factor, the variation between them is the *real variation*, and it is interpreted as the absolute value of the difference, or the measurements are ordered in the differential to yield a positive result.

17.5.4. If in the second measurement the factor has obtained a value higher than in the first measurement, then the first measurement plus the real variation equals the second measurement.

17.5.5. If in the second measurement the factor has obtained a value lower than in the first measurement, then the first measurement minus the real variation equals the second measurement.

17.5.6. *Real variation*—i.e., the variation that has actually occurred in a factor between two measurements—is therefore the variation that has occurred between a first and a second measurement. Thus, if in growth the second measurement equals the sum of the first measurement plus the real variation, logically what should be forecast for a future measurement is that—if this growth rate continues—necessarily the future measurement, the *Forecasted Probability*, would equal the sum of the second measurement plus the real variation, with the real variation in growth being equal to the difference of the second measurement minus the first measurement.

17.5.7. In decrease studies—where the second measurement is lower than the first—the real variation is the variation that has actually occurred from the first to the second

measurement in decrease. Then second measurement equals first measurement minus real variation; with the real variation equal to the difference of the first minus the second measurement. Therefore, if this rate of decrease continues, one would expect the future measurement to be equal to the second measurement minus the real variation—i.e., the *Forecasted Probability* in a decrease study.

17.5.8. *Projected Probability* is a plan for what the result of the future measurement should be in a project of ideal trend—from the first measurement to the realization of the ideal—product of the idealization of reality. Mathematics is the idealization of reality for the realization of the ideal.

17.5.9. *Forecasted Probability* is the forecast of what the result of the future measurement should be, based on the forecast of the *real trend*, the real trend being the real variation between two different measurements, in order to determine what the true real realization of the ideal is, to critique the real trend toward the ideal.

17.5.10. To the sum (in growth) of second measurement plus real variation, or the subtraction (in decrease) of second measurement minus real variation, error margins can be added—within which one is willing to accept a future measurement. The error margins in *Forecasted Probability* will depend on whether it is a study of growth or of decrease.

17.5.10.1. In a growth study, the critical probability as an error margin will equal the product of the error percentage X (out of one hundred) and the difference between one and the *Forecasted Probability* itself.

Error margin in Forecasted Probability (growth):

$$\pm p(xc) = [1 - (p(xi/2nd) + VR)] \cdot (X : 100)$$

$$(p(xi/2nd) + VR) + p(xc) = \text{upper limit of the error margin}$$

$$(p(xi/2nd) + VR) - p(xc) = \text{lower limit of the error margin}$$

VR = Real Variation
 [Real Variation, Spanish]

$p(xi/2nd)$ = empirical probability, second measurement

X = error percentage

17.5.10.2. In a decrease study, since the factor to be decreased is the *Forecasted Probability* itself, necessarily the error margin will equal the product of the error percentage X (out of one hundred) and the *Forecasted Probability* itself.
Error margin in Forecasted Probability (decrease):

$$\pm p(xc) = (p(xi/2nd) - VR) \cdot (X : 100)$$

$$(p(xi/2nd) - VR) + p(xc) = \text{upper limit of the error margin}$$

$$(p(xi/2nd) - VR) - p(xc) = \text{lower limit of the error margin}$$

VR = Real Variation
 [Real Variation, Spanish]

$p(xi/2nd)$ = empirical probability, second measurement

X = error percentage

17.5.11. If the second measurement has a sample N different from the sample N of the first measurement, then apply the *effect of N* to the first measurement in order to determine the real variation based on the difference—in *growth*—of the second measurement minus the product of the first measurement times the effect of N. In *decrease*, with samples of different N, real variation equals the difference of the product of the first measurement times the effect of N minus the second measurement. In any case, in this particular case the second measurement is designated measurement “m,” and the first measurement is designated measurement “mi.”

17.5.12. *Forecasted Probability* will be symbolized by “ $p(x_i)''$,” i.e., empirical probability double-prime. The reason *Forecasted Probability* is symbolized “ $p(x_i)''$ ” (empirical probability double-prime) is that it is obtained from *two* measurements, unlike *Projected Probability*, which is obtained from a *single* measurement; hence *Projected Probability* is symbolized as empirical probability prime “ $p(x_i)'$.”

$p(x_i)'' = \text{Forecasted Probability, from real variation of two measurements}$

$p(x_i)' = \text{Projected Probability, from Reliable Possible Variation of the first measurement}$

17.5.13. Contrasts between Forecasted Probability and the results of future measurements

17.5.13.1. In principle, any future measurement within the forecasted error margins is accepted. In a growth study, one may accept any future measurement equal to or greater than the lower limit of the error margin, or—if stricter—only those future measurements equal to or greater than the *upper* limit of the error margin. In a decrease study, one may accept any future measurement equal to or less than the *upper* limit of the error margin, and—if stricter—only those equal to or less than the *lower* limit of the error margin.

17.5.13.2. If the future measurements are obtained on samples whose N differs from the N of the previous measurements—since the second measurement has been designated “ m ”—apply the *effect of N* to the future measurement, where the effect of N equals the quotient of the N magnitude of the future measurement “ m_i ” over the N magnitude of the second measurement “ m .” Thus, in this particular case, the future measurement is designated “ m_i ,” and the second measurement “ m .” The effect of N times the empirical probability of the future measurement equals the product of the future measurement by the quotient of its N magnitude over the N magnitude of the second measurement.

17.5.13.3. Contrasts in Forecasted Probability, in any case, must be formulated within rational logic; that is, if a Forecasted Probability equals a number that is, in a certain sense, irrational for the logic of statistical probability, possible contrasts will not be pertinent. Since it is possible that a Forecasted Probability might exceed one or be negative in sign, this is irrational: an empirical probability cannot exceed one, because the part can never be greater than the whole—at most, the part can itself be a whole (a sample or population is itself a whole while also being, externally, part of the universe). Therefore, empirical probabilities do not exceed one. Likewise, the empirical probability of a positive event cannot have a negative sign. If a *Forecasted Probability* is greater than one, one will simply say that the *Forecasted Probability* is the *Maximum Possible Empirical Probability*, below which a lower error-margin limit may be incorporated; similarly, if a *Forecasted Probability* is less than zero (negative), one will simply say that the *Forecasted Probability* is the *Minimum Possible Empirical Probability*, above which an upper error-margin limit may be incorporated.

17.5.13.4. If a *Forecasted Probability* exceeds one, the *Maximum Possible Probability* (one) is forecast, and future measurements will be contrasted based on a limit within the error margin: error percentage X out of one hundred. Any empirical probability equal to or greater than the difference between one and the error margin would be accepted.

17.5.13.5. If a *Forecasted Empirical Probability* is less than zero, the *Minimum Possible Empirical Probability* (zero) is forecast, and future measurements will be contrasted based on an error-margin limit: error percentage X out of one hundred. Any empirical probability equal to or less than the error margin above zero is accepted.

17.5.13.6. In those sciences that use mathematical language and allow probabilistic magnitudes less than zero or greater than one—percentages above one hundred percent or below zero—then, provided it is logically rational for those sciences, forecasts above one or below zero may be possible, as long as they are rational within that science. For example, in economics terms above one hundred percent or below zero percent are used continually; whenever it is logical in a science to forecast above one hundred percent or below zero percent, such forecasts can be made within the logic of those particular sciences that do allow it.

17.5.13.7. In any case and regardless of any other study criterion, for linearly forecast prediction models to be truly useful, one must always respect the linear logic of the forecast. Insofar as future measurements are forecast based on *real trends*, independently of ideals, whenever in a second measurement—whatever the object of study—the second measurement is greater than the first, it will be said that there has been *growth*, and the ordering of factors in the real variation—so that it is realistic with respect to the real data—is the *difference of the second measurement minus the first*, regardless of any other factor or criterion, if one truly wishes to know *a priori* the real trend to be verified *a posteriori*, even if that real trend established *a priori* does not conform to the ideals. Since, whatever the ideals, it is necessary regarding the real trend to know at least what the model's real trend is—whether or not it matches the initial ideals; likewise, regardless of any other factor or criterion, if a second measurement is verified to be lower than the first, the calculation of real variation will be *equal to the second measurement minus the first measurement*. Thus, it is on that rate of decrease—whether or not it fits our ideals—that the model's real trend must be predicted, in linear forecast form, whether or not it conforms to the ideals established *a priori*.

17.5.13.8. *Independently of the ideals of scientific policy, once a trend of growth (second measurement greater than the first) or of decrease (second measurement less than the first) is recognized—whether or not it corresponds to a priori ideals—scientific policy must make a realistic forecast of future trends, whether or not they align with the ideals, in order to have a positive grasp of the real trend, apart from the ideal trend. Thus, only by knowing reality positively can scientific policy later intervene directly in reality to realize the ideal in reality.*

17.5.14. Forecasted Probability for a third measurement

17.5.14.1. Forecasted Probability for a third measurement when the two measurements have the same N

17.5.14.1.1. Forecasted Probability for a third measurement, with the two measurements having the same N, in a growth study:

$$p(xi)'' = \{ p(xi/2nd) + [p(xi/2nd) - p(xi/1st)] \} \pm p(xc)$$

$$\pm p(xc) = \{ 1 - \{ p(xi/2nd) + [p(xi/2nd) - p(xi/1st)] \} \} \cdot (X : 100)$$

X = error percentage

17.5.14.1.2. Forecasted Probability for a third measurement, with the two measurements having the same N, in a decrease study:

$$p(xi)'' = \{ p(xi/2nd) - [p(xi/1st) - p(xi/2nd)] \} \pm p(xc)$$

$$\pm p(xc) = \{ p(xi/2nd) - [p(xi/1st) - p(xi/2nd)] \} \cdot (X : 100)$$

X = error percentage

17.5.14.2. Forecasted Probability for a third measurement when the first measurement has an N different from the second measurement

17.5.14.2.1. Forecasted Probability for a third measurement, when the first measurement has an N different from the second measurement, in a growth study:

$$p(x_i)' = \{ p(x_i/2nd-m) + [p(x_i/2nd-m) - (p(x_i/1st-mi) \cdot (N_{mi} : N_m))] \} \pm p(x_c)$$

$$\pm p(x_c) = \{ 1 - \{ p(x_i/2nd-m) + [p(x_i/2nd-m) - (p(x_i/1st-mi) \cdot (N_{mi} : N_m))] \} \} \cdot (X : 100)$$

X = error percentage

17.5.14.2.2. *Forecasted Probability* for a third measurement, when the first measurement has an N different from the second measurement, in a decrease study

$$p(x_i)' = \{ p(x_i/2nd-m) - [(p(x_i/1st-mi) \cdot (N_{mi} : N_m)) - p(x_i/2nd-m)] \} \pm p(x_c)$$

$$\pm p(x_c) = \{ p(x_i/2nd-m) - [(p(x_i/1st-mi) \cdot (N_{mi} : N_m)) - p(x_i/2nd-m)] \} \cdot (X : 100)$$

X = error percentage

17.5.15. Forecasted Probability for an nth measurement

17.5.15.1. Concept of Forecasted Probability for an nth measurement

17.5.15.1.1. *Forecasted Probability* for nth measurements, based on the real variation between two measurements, will equal, in growth, the sum of the second measurement plus the product of the real variation by (nth measurement minus two). Obviously, we subtract two from the nth index because the nth measurement is calculated from the two previous measurements. If the real variation is obtained from two measurements with different N, the first measurement is usually designated “mi,” so the effect of N will equal the first measurement times the quotient of its N magnitude over the N magnitude of the second measurement.

17.5.15.1.2. In a decrease study, from the second measurement subtract the value resulting from the product of (nth measurement minus two) by the real variation of the two measurements. If the two measurements from which the real variation is deduced have different N, the effect of N must be estimated on the first measurement by multiplying it by the quotient of its N magnitude over the N magnitude of the second measurement.

17.5.15.1.3. The nth *Forecasted Probability* is symbolized “ $p(x_i/me)^{''}$ ”.

$p(x_i/me)^{''}$ = nth Forecasted Probability

me= *medicion emesima* (Spanish), nth measurement

17.5.15.2. Forecasted Probability for an nth measurement

17.5.15.2.1. *Forecasted Probability* for an nth measurement, when the two measurements have the same N, in a growth study:

$$p(xi/me)^{''} = \{ p(xi/2nd) + \{ [p(xi/2nd) - p(xi/1st)] \cdot (me - 2) \} \} \pm p(xc)$$

$$\pm p(xc) = \{ 1 - \{ p(xi/2nd) + \{ [p(xi/2nd) - p(xi/1st)] \cdot (me - 2) \} \} \} \cdot (X : 100)$$

X = error percentage

17.5.15.2.2. *Forecasted Probability* for an nth measurement, when the two measurements have the same N, in a decrease study:

$$p(xi/me)^{''} = \{ p(xi/2nd) - \{ [p(xi/1st) - p(xi/2nd)] \cdot (me - 2) \} \} \pm p(xc)$$

$$\pm p(xc) = \{ p(xi/2nd) - \{ [p(xi/1st) - p(xi/2nd)] \cdot (me - 2) \} \} \cdot (X : 100)$$

X = error percentage

17.5.15.3. Forecasted Probability for an nth measurement when the first measurement has an N different from the second measurement

17.5.15.3.1. *Forecasted Probability* for an nth measurement, when the first measurement has an N different from the second measurement, in a growth study:

$$p(x_i/me)^{\prime\prime} = \{ p(x_i/2nd-m) + \{ [p(x_i/2nd-m) - (p(x_i/1st-mi) \cdot (N_{mi} : N_m))] \cdot (me - 2) \} \} \pm p(x_c)$$

$$\pm p(x_c) = \{ 1 - \{ p(x_i/2nd-m) + \{ [p(x_i/2nd-m) - (p(x_i/1st-mi) \cdot (N_{mi} : N_m))] \cdot (me - 2) \} \} \cdot (X : 100)$$

X = error percentage

17.5.15.3.2. *Forecasted Probability* for an nth measurement, when the first measurement has an N different from the second measurement, in a decrease study:

$$p(x_i/me)^{\prime\prime} = \{ p(x_i/2nd-m) - \{ [(p(x_i/1st-mi) \cdot (N_{mi} : N_m)) - p(x_i/2nd-m)] \cdot (me - 2) \} \} \pm p(x_c)$$

$$\pm p(x_c) = \{ p(x_i/2nd-m) - \{ [(p(x_i/1st-mi) \cdot (N_{mi} : N_m)) - p(x_i/2nd-m)] \cdot (me - 2) \} \} \cdot (X : 100)$$

X = error percentage

17.6. Forecasted Dispersion

17.6.1. If the real variation between two factors is the difference between those two factors, then—given two measurements—the real variation between the two measurements is the difference of the two measurements. If in a first measurement a certain Mean or Standard Deviation (DMoS) has been obtained, and in a second measurement a different Mean or Standard Deviation has been obtained, the difference between the two Means or Standard Deviations is the real variation experienced by the Mean or Standard Deviation between the two measurements.

17.6.2. In a bias study, the object is to increase the Mean or Standard Deviation; therefore, if an increase from the first measurement's Mean or Standard Deviation to the second measurement's Mean or Standard Deviation is indeed verified, the real variation will equal the difference of the second measurement's Mean or Standard Deviation minus the first measurement's Mean or Standard Deviation. It is then forecast that, for a future measurement, the Mean or Standard Deviation of the future measurement will be equal to the sum of the second measurement's Mean or Standard Deviation plus the observed real variation (i.e., the difference between the second measurement's Mean or Standard Deviation and the first's). In any case, the result of the sum equals the *Forecasted Mean or Standard Deviation*, in a bias study, for a future measurement—to which error margins can be added.

17.6.3. In equality of opportunity, if the real variation of the second measurement's Mean or Standard Deviation is less than the first measurement's Mean or Standard Deviation, what is to be expected is that—if that rate of increase in equality of opportunity continues—the *Forecasted Mean or Standard Deviation* for a future measurement will equal the difference of the second measurement's Mean or Standard Deviation minus the real variation observed (i.e., the difference between the first measurement's Mean or Standard Deviation and the second's). An error margin can be added to the result of the subtraction, yielding the *Forecasted Mean or Standard Deviation* in equality of opportunity for a future measurement. To make normal forecasts, use normal dispersion (i.e., normal Mean or Standard Deviation) as normal projections of sample dispersion.

17.6.4. The error margin in the *Forecasted Mean Deviation* will differ depending on whether it is a bias study or an equality-of-opportunity study; in any case, normal predictions will be made.

17.6.4.1. The error margin in *Forecasted Mean Deviation*, in a bias study—since the examples will follow normal Mean Deviation—will equal the product of the error percentage X (out of one hundred) and the difference between the *Maximum Theoretical Possible Mean Deviation* and the result of the sum of the normal Mean Deviation of the second measurement plus the real variation, with real variation equal to the difference between the normal Mean Deviation of the second measurement and that of the first.

Error margin in Forecasted Mean Deviation in bias:

$$\pm p(xc) = \{ \{ [(1 - 1/N) \cdot 2] : N \} - (DM/2nd + VR) \} \cdot (X : 100)$$

$$(DM/2nd + VR) + p(xc) = \text{upper limit of the error margin}$$

$$(DM/2nd + VR) - p(xc) = \text{lower limit of the error margin}$$

DM/2nd = Mean Deviation, second measurement

VR = real variation [*Real Variation*, Spanish] = DM/2nd - DM/1st

DM/1st = Mean Deviation First Measurement

X = error percentage

17.6.4.2. The error margin in *Forecasted Mean Deviation*, in equality of opportunity, will be the product of the error percentage X (out of one hundred) and the difference of the normal Mean Deviation of the second measurement minus the real variation, with real variation equal to the difference between the normal Mean Deviation of the first measurement and that of the second measurement.

Error margin in Forecasted Mean Deviation in equality of opportunity:

$$\pm p(xc) = (DM/2nd - VR) \cdot (X : 100)$$

$$(DM/2nd - VR) + p(xc) = \text{upper limit of the error margin}$$

$$(DM/2nd - VR) - p(xc) = \text{lower limit of the error margin}$$

DM/2nd = Mean Deviation, second measurement

VR = real variation [*Real Variation*, Spanish] = DM/1st - DM/2nd

X = error percentage

17.6.5. If, to establish the real variation, there are differences in the N magnitude of the second measurement and the N magnitude of the first measurement, the effect of N must be applied to the Mean or Standard Deviation (DMoS) of the first measurement, which in this particular case is designated “mi,” while the second measurement is always designated “m.”

17.6.6. *Interpretation of the Forecasted Mean or Standard Deviation*

17.6.6.1. In principle, any value obtained in a future measurement that lies within the error margin of the *Forecasted Mean or Standard Deviation* will be accepted. In a bias study, one may accept any future measurement’s Mean or Standard Deviation that is equal to or greater than the *lower limit* of the error margin, or—if stricter—accept only those Means or Standard Deviations equal to or greater than the *upper* limit of the error margin. In a study of equality of opportunity, one may accept any future measurement’s Mean or Standard Deviation that is equal to or less than the *upper* limit of the error margin, or—if stricter—only those Means or Standard Deviations equal to or less than the *lower* limit of the error margin.

17.6.6.1.1. If a Forecasted Mean or Standard Deviation is greater than the Maximum Theoretical Possible Mean Deviation or greater than the Maximum Theoretical Possible Standard Deviation—since a Mean or Standard Deviation above the theoretical maximum would be irrational—one will simply say that the Maximum Theoretical Possible Mean Deviation or the Maximum Theoretical Possible Standard Deviation is forecast. Below that value an error margin can be established on which to carry out the rational critique of the future measurement’s Mean or Standard Deviation.

17.6.6.1.2. Likewise, if a Forecasted Mean or Standard Deviation were negative, since it is irrational for positive empirical probabilities to have a negative dispersion, one will simply say that the forecast is Mean or Standard Deviation equal to zero. Above that, an error margin can be established to contrast the future measurement’s Mean or Standard Deviation against the error margin above zero dispersion as decided by scientific policy.

17.6.6.1.3. In any case, regardless of the object of study, insofar as the forecast is made on *observed real trends*, whatever the ideal may be, whenever the first measurement's Mean or Standard Deviation is higher than the second's, the real variation will be the difference of the first measurement's dispersion minus the second's, which constitutes a forecast of an increase in equality of opportunity in the sample. Conversely, if the second measurement shows an increase in dispersion, then real variation equals the difference of the second measurement's dispersion minus the first's, which constitutes a forecast of an increase in bias—regardless of initial ideals. Based on these real rates of dispersion, the **forecasted dispersion for bias or equality is calculated according to the real trend.** If the second measurement's Mean or Standard Deviation equals that of the first, i.e., the dispersion is the same in both measurements, then there has been no real variation in dispersion and zero variation in dispersion would be forecast for subsequent measurements—whether or not real variations or forecasts fit our ideals.

17.6.6.1.4. Whether or not future real trends align with initial ideals, that—and no other—is the future reality that may occur, and it is **upon this true real reality that scientific policy must intervene directly to achieve the ideals. All the more reason to obtain the most positive knowledge possible of the true future reality, so that, based on the most realistic forecasts of the future, one can later intervene objectively to realize the ideals in true reality.**

17.6.6.2. If the future measurement has a sample N different from the sample N of the second measurement, the second measurement will always be measurement “ m ,” and the future measurement, in this particular case, will be measurement “ m_i .” The effect of N is multiplied by the future measurement's Mean or Standard Deviation, where the effect of N equals the quotient of the N magnitude of measurement “ m_i ” (in this case, the future measurement “ m_i ”) divided by the N magnitude of measurement “ m ” (in this case, the second measurement “ m ”).

17.6.7. Since *Impossible Probability* gives priority to Mean Deviation [DM], examples will be provided using *Forecasted Mean Deviation*. In any case, to adapt the Second Method to the First Method, one can always replace *normal Mean Deviation* with *normal Standard Deviation*, and replace the *Maximum Theoretical Possible Mean Deviation* with the *Maximum Theoretical Possible Standard Deviation*.

17.6.7.1. The *Forecasted Mean Deviation* will be symbolized by “DM´´” (normal Mean Deviation double-prime) [*Desviacion media prima prima*]. If *Forecasted Standard Deviation* is computed, it will be symbolized by “S´´” (normal Standard Deviation double-prime). Generically, *Forecasted Mean or Standard Deviation* will be symbolized “DMoS´´”.

DM´´ = Forecasted Mean Deviation

S´´ = Forecasted Standard Deviation

DMoS´´ = Forecasted Mean or Standard Deviation

17.6.8. Forecasted Mean Deviation for a third measurement

17.6.8.1. Forecasted Mean Deviation for a third measurement when the first and second measurements have the same N

17.6.8.1.1. Forecasted Mean Deviation for a third measurement, when the first and second measurements have the same N, in the case of *increasing bias* between the two measurements:

$$DM´´ = \{ DM/2nd + (DM/2nd - DM/1st) \} \pm p(xc)$$

$$p(xc) = \{ \{ [(1 - 1/N) \cdot 2] : N \} - \{ DM/2nd + (DM/2nd - DM/1st) \} \} \cdot (X : 100)$$

X = error percentage

DM/2nd= Mean Deviation Second Measurement

DM/1st= Mean Deviation First Measurement

17.6.8.1.2. Forecasted Mean Deviation for a third measurement, when the first and second measurements have the same N, in the case of *increasing equality of opportunity* between the two measurements:

$$DM'' = \{ DM/2nd - (DM/1st - DM/2nd) \} \pm p(xc)$$

$$p(xc) = \{ DM/2nd - (DM/1st - DM/2nd) \} \cdot (X : 100)$$

X = error percentage

DM/2nd= Mean Deviation Second Measurement

DM/1st= Mean Deviation First Measurement

17.6.8.2. Forecasted Mean Deviation for a third measurement when the first and second measurements have different N

17.6.8.2.1. Forecasted Mean Deviation for a third measurement, when the first and second measurements have different N, in the case of *increasing bias* between the two measurements:

$$DM'' = \{ DM/2nd-m + [DM/2nd-m - (DM/1st-mi \cdot (Nmi : Nm))] \} \pm p(xc)$$

$$p(xc) = \{ \{ [(1 - 1/Nm) \cdot 2] : Nm \} - \{ DM/2nd-m + [DM/2nd-m - (DM/1st-mi \cdot (Nmi : Nm))] \} \} \cdot (X : 100)$$

X = error percentage

DM/2nd= Mean Deviation Second Measurement

DM/1st= Mean Deviation First Measurement

17.6.8.2.2. Forecasted Mean Deviation for a third measurement, when the first and second measurements have different N, in the case of *increasing equality of opportunity* between the two measurements:

$$DM'' = \{ DM/2nd-m - [(DM/1st-mi \cdot (Nmi : Nm)) - DM/2nd-m] \} \pm p(xc)$$

$$p(xc) = \{ DM/2nd-m - [(DM/1st-mi \cdot (Nmi : Nm)) - DM/2nd-m] \} \cdot (X : 100)$$

X = error percentage

DM/2nd= Mean Deviation Second Measurement

DM/1st= Mean Deviation First Measurement

17.6.9. Forecasted Mean Deviation for an nth measurement

The way to symbolize the *Forecasted Mean Deviation* for an nth measurement will be

“DM(me)''”.

DM(me)'' = nth Forecasted Mean Deviation

me= nth measurement [medicion emesima, Spanish]

17.6.9.1. Forecasted Mean Deviation for an nth measurement when the first and second measurements have the same N

17.6.9.1.1. Forecasted Mean Deviation for an nth measurement, when the first and second measurements have the same N, in the case of *increasing bias* between the two measurements:

$$DM(me)'' = \{ DM/2nd + \{ (DM/2nd - DM/1st) \cdot (me - 2) \} \} \pm p(xc)$$

$$p(xc) = \{ \{ [(1 - 1/N) \cdot 2] : N \} - \{ DM/2nd + \{ (DM/2nd - DM/1st) \cdot (me - 2) \} \} \} \cdot (X : 100)$$

X = error percentage

DM/2nd= Mean Deviation Second Measurement

DM/1st= Mean Deviation First Measurement

17.6.9.1.2. Forecasted Mean Deviation for an nth measurement, when the first and second measurements have the same N, in the case of *increasing equality of opportunity* between the two measurements:

$$DM(me)'' = \{ DM/2nd - \{ (DM/1st - DM/2nd) \cdot (me - 2) \} \} \pm p(xc)$$

$$p(xc) = \{ DM/2nd - \{ (DM/1st - DM/2nd) \cdot (me - 2) \} \} \cdot (X : 100)$$

X = error percentage

DM/2nd= Mean Deviation Second Measurement

DM/1st= Mean Deviation First Measurement

17.6.9.2. Forecasted Mean Deviation for an nth measurement when the first and second measurements have different N

17.6.9.2.1. Forecasted Mean Deviation for an nth measurement, when the first and second measurements have different N, in the case of *increasing bias* between the two measurements:

$$DM(me)^{''} = \{ DM/2nd-m + \{ [DM/2nd-m - (DM/1st-mi \cdot (Nmi : Nm))] \cdot (me-2) \} \} \pm p(xc)$$

$$p(xc) = \{ \{ [(1 - 1/Nm) \cdot 2] : Nm \} -$$

$$\{ DM/2nd-m + \{ [DM/2nd-m - (DM/1st-mi \cdot (Nmi : Nm))] \cdot (me - 2) \} \} \cdot (X : 100)$$

X = error percentage

DM/2nd= Mean Deviation Second Measurement

DM/1st= Mean Deviation First Measurement

17.6.9.2.2. Forecasted Mean Deviation for an nth measurement, when the first and second measurements have different N, in the case of *increasing equality of opportunity* between the two measurements:

$$DM(me)'' = \{ DM/2nd-m - \{ [(DM/1st-mi \cdot (Nmi : Nm)) - DM/2nd-m] \cdot (me-2) \} \} \pm p(xc)$$

$$p(xc) = \{ DM/2nd-m - \{ [(DM/1st-mi \cdot (Nmi : Nm)) - DM/2nd-m] \cdot (me - 2) \} \} \cdot (X:100)$$

X = error percentage

DM/2nd= Mean Deviation Second Measurement

DM/1st= Mean Deviation First Measurement

17.7. Contrasts between projected values and forecasted values

17.7.1. Projected values are projected on the basis of a *Reliable Possible Variation* toward the ideal trend; forecasted values are values predicted on the basis of a *real variation* [In other words in GAI terms, forecasted values are second stage Modelling System, projected values are second stage Decisional System, the point in the sixth phase is to build a share space where to merge models and projects, the global plan, the Global Project System, GPS. Obviously, Modelling System and Decisional will utilize all forms of mathematical analysis available for its construction, very especially in the design of our GPS].

17.7.2. Both projected values and forecasted values are, in form, possible predictions: based on a *Reliable Possible Variation* toward an ideal, it is predicted that a factor should be projected to a given value; based on a *real variation*, it is predicted that a factor should reach a given forecasted value.

17.7.3. Prediction—whether of an ideal project or a real forecast—is always an *a priori* estimate about a possible future occurrence.

17.7.4. Only within the accepted error margin does a meaning attain propositional reliability; beyond the accepted error, reliability will be insufficient. Hence, outside the error margin, an idea may indeed be an idea produced by brain *physiology*, but not identical to real reality; outside the error margin, a possible reality may be a possible reality, but insofar as it is not identical to any sufficiently reliable idea, it is, for the time being, an unknown reality. At present, realities can only be known through the physiological work of the brain. If a given reality lacks a reliable mental representation, it may be truly real and yet be completely unknown; a reality that is not, for the moment, identical to a true idea cannot be a known reality. A reality not yet known—as possible knowledge—is inevitably knowable in due time, provided there is sufficient or infinite time. **Truth is a matter of time.** And even if accepted as reliable, it will be reliable within rational time—therefore provisionally reliable; beyond rational time, everything lacks reliability.

17.7.5. Within the *a priori* accepted error margin, one may project *a priori* an ideal value for a second measurement and yet, in the second measurement, the value measured may differ from the projected value. A real value in the second measurement that differs from the projected value is a real non-ideal value. The non-necessary identity between *a priori* idealism and *a posteriori* real reality explains the need for the error margin: the smaller the error margin in the linear projection, the stricter the identity required between the real and the ideal; the smaller the error margin, the greater the likelihood of rejecting empirical hypotheses not identical to the ideal project. The larger the error margin, the greater the likelihood of accepting that an empirical hypothesis is identical to the ideal. However, if universal acceptance of an empirical hypothesis is set on too large an error margin, the probability of universal propositional reliability—relative to space-time—decreases as a larger error margin is accepted, because the greater the accepted error margin, the shorter the time needed for the inevitability of the accepted error to materialize. An unreliable theory becomes erroneous in a short time. **Truth is relative because it is a matter of time.** A very strict error margin may accept fewer empirical hypotheses—since the probability that a real trend is identical to the ideal

trend is lower—but while the number of accepted empirical hypotheses decreases as the error margin is more critical, those empirical hypotheses accepted on a more critical error margin are propositions whose propositional reliability will endure for a longer period of time, since the smaller the accepted error margin, the longer the time possibly needed for the inevitability of error to become possible.

17.7.6. A second measurement may not be identical to the ideal value projected for the second measurement; but in any case, that real value obtained for the second measurement is the real value of that second measurement. Based on the real variation between the first and second measurements, real forecast values can be established, and real forecasts about the real trend in future measurements can be made.

17.7.7. A real value in a second measurement may be identical to the ideal value projected for the second measurement, insofar as the real variation between the two measurements is similar to the *Reliable Possible Variation* in the ideal trend.

17.7.8. In the particular case where the real value of the second measurement is not identical to the ideal projection, this does not necessarily imply that all future *n*th measurements must be different from the values projected.

17.7.9. Likewise, even if the real value of the second measurement is identical to the ideal projected value, this is not sufficient to justify that all future *n*th measurements must necessarily be real *n*th measurements identical to the ideal *n*th projections.

17.7.10. It may be the case that, for a given factor, the real value of the second measurement is not identical to the ideal, yet in future measurements that same factor may produce real *n*th measurements identical to the ideal *n*th projections for those future measurements.

17.7.11. The fact that, for a given factor, the second measurement's real value is identical to the ideal does not imply that this same factor will really tend to be ideal indefinitely; nor does it imply that if this same factor had, in the second measurement, a real value not identical to the ideal, then necessarily, in future *n*th measurements, it would indefinitely be a non-ideal real measurement. It may occur—dialectically—that factors not truly ideal in the second measurement nonetheless become identical to the

ideals projected for future n th measurements; and it may occur that factors truly identical to the ideal in the second measurement later yield real n th measurements that are not equal to the ideal projected for that n th measurement.

17.7.12. A factor in a second measurement may, then, have a real measurement identical to or different from the ideal, and this does not imply that the behavior observed in the second measurement—being identical or different from the ideal—will be a regular or infinite pattern of behavior for all future n th measurements.

17.7.13. Insofar as the real variation between two measurements does not signify a regular or infinite behavior pattern, one can forecast a value for future n th measurements based on a real variation; and, based on the forecasted value for each n th measurement, determine whether the factor's real measurement in each future n th measurement is identical to the value forecast from the real variation.

17.7.14. In fact, over a set of M measurements, the circumstance that in a third measurement the real value equals the forecasted value for that third measurement does not require that, for all future real measurements, the result must always be identical to the forecasted value for the future measurements.

17.7.15. It may be the case that a real value in the third measurement is not identical to the forecasted value, and yet that same factor later has real measurements identical to the forecasted values in future measurements.

17.7.16. Thus, linear projection—whether *Projected Probability* or *Projected Mean or Standard Deviation*—and linear forecast—whether *Forecasted Probability* or *Forecasted Mean or Standard Deviation*—can provide sufficient elements for the rational critique of the behavior of subjects or options in real reality in the future, by establishing ideal or real *a priori* bases on which to critique future reality *a posteriori*.

17.7.17. Ideally, in any case, from a second measurement onward, across the entire sequence of subsequent measurements, the real measurements would tend to distribute around the ideals for each future measurement, such that the ideal projected value and the real forecasted value tend to identify with one another. In a true ideal of linear equations—the tendency toward identity of the real and the ideal within the

accepted error margin—in linear projection and forecast, the ideal model of rational critique will normally be the *critical probability*.

18. The statistical ranking of empirical probabilities

18.1. In probability studies, the way the trend can be a variable dependent on the sample—whether sample N or direct scores/frequencies, depending on the type of universe—can be a determining factor. However, the position each nth position (“ni”) occupies, in itself, initially determines nothing unless the sample is logically ordered spatiotemporally, in discrete categories, or in a ranking. In samples not logically ordered, the position “ni” would only be of interest if “ni” is ordered by growth or decrease, by arranging subjects or options in an order of precedence, in a ranking. The nth position in the ranking that a subject or option may occupy will be symbolized by “n^o”.

n^o = nth position in the ranking

18.1.1. Ordering in precedence of “n^o” by empirical probability

18.1.1.1. Ordering in precedence, in nth positions in the ranking (“n^o”), in growth measurements. In growth studies, ordering the “n^o” by empirical probability is done so that the first subject or option is the one with the lowest empirical probability (the minimum empirical probability), and the last subject or option is the one with the highest empirical probability (the maximum empirical probability), ordering from the first to the nth subject or option according to its empirical probability, such that any subject or option has an empirical probability greater than that of the preceding subject or option, and any subject or option has an empirical probability less than that of the following subject or option. The only exceptions are the first subject or option, which will have no lower predecessor, and the last subject or option, which will have no higher successor.

Ordering in precedence in growth:

n^0 = nth position in the ranking

$n^01 = p(x1) \rightarrow$ empirical probability of position number one in the ranking

$n^02 = p(x2) > p(x1) \rightarrow$ empirical probability of position number two in the ranking

$n^03 = p(x3) > p(x2) \rightarrow$ empirical probability of position number three in the ranking

$n^0 = p(xn) > p(x(n - 1)) \rightarrow$ in growth, each one greater than the previous

18.1.1.1.1. The ordering in precedence in growth will therefore be such that each subject or option has an empirical probability greater than that of the previous one, except for the first subject or option, which has no predecessor.

18.1.1.1.2. Ordering in precedence of “ n^0 ” in decrease studies. In decrease studies, subjects or options can be ordered in precedence by empirical probability in descending form; that is, the first subject or option will have an empirical probability greater than that of the following subject or option, and as a general rule, every subject or option will have an empirical probability lower than that of the previous subject or option; thus, every subject or option will have an empirical probability higher than that of the subsequent subject or option. In this way, the first subject or option will have the maximum empirical probability, while the last (nth) subject or option will have the minimum empirical probability. Thus, only the first subject or option will have no higher

predecessor—being first—and only the last (nth) subject or option will have no lower successor—being last.

Ordering in precedence in decrease:

n^0 = nth position in the ranking

$n^01 = p(x1) \rightarrow$ empirical probability of position number one

$n^02 = p(x2) < p(x1) \rightarrow$ empirical probability of position number two

$n^03 = p(x3) < p(x2) \rightarrow$ empirical probability of position number three

$n^0 = p(xn) < p(x(n - 1)) \rightarrow$ in decrease, each one lower than the previous

18.1.1.3. Thus, the ordering in precedence, in decrease, will be such that each subject or option has an empirical probability lower than that of the previous one, except for the first subject or option, which will have no predecessor.

18.1.2. The ranking of the order of precedence: In each measurement, every subject or option will be ordered in precedence to its position “ n^0 ,” whether in growth or in decrease, as corresponds in each measurement, according to precedence by empirical probability. In this way, a subject or option may move within the ranking of orderings, according to changes that have occurred either in itself or in the sample as a whole. The fact that the same subject or option maintains in a future measurement the direct score or frequency from the first measurement does not guarantee that it will have the same place in the ranking in the future measurement. Directly, the empirical probability of any subject or option in a future measurement can change due to changes experienced by

that same subject or option. Indirectly, any subject or option can experience changes in empirical probability because of changes in other subjects or options that alter the sum of direct scores or frequencies, automatically producing changes in all empirical probabilities. The fact that a subject or option keeps its direct score or frequency intact in future measurements is not sufficient to preserve its position in the ranking, since indirectly it may be affected by direct changes in other subjects or options that can alter the ranking's original order.

18.1.3. Ordering by precedence, ranking, and the value of “ n^o ”

18.1.3.1. In growth or decrease studies of empirical probabilities, the n^{th} position “ n^i ” that a subject or option occupies does not necessarily have any influence on determining the empirical probability.

18.1.3.2. If we arrange the subjects or options in an order of precedence, then we construct a ranking, and within the ranking the “ n^o ” position does matter, because the “ n^o ” (n^{th} position in the ranking) indicates the position, place, or slot that the subject or option occupies within the sample's ranking.

18.1.3.3. By ordering subjects or options according to their n^{th} position in the ranking (“ n^o ”), the ranking allows us to obtain information about a subject or option based on its position in the ranking. The value “ n^o ” will symbolize the n^{th} position that the subject or option occupies in the ranking and provides sufficient information to locate the subject or option within the sample's ranking.

18.1.3.4. Ordering by precedence in the growth and decrease ranking

18.1.3.4.1. In decreasing samples, the order of precedence in the ranking will be descending. A descending ranking means that each subject or option is ordered so that its empirical probability is lower than that of the preceding subject or option and higher than that of the following subject or option. The first subject or option will have no predecessor and will have the highest (maximum) empirical probability in the sample; the second subject or option will have an empirical probability lower than the first subject or option and higher than the rest, and so on, ordering all subjects or options in descending fashion until reaching the last subject or option, which will have the lowest

(minimum) empirical probability in the entire sample. The n th positions of the sample's ranking will be ordered from highest to lowest according to each subject's or option's empirical probability.

18.1.3.4.2. In increasing samples, the order of precedence in the ranking will be ascending. This implies that any subject or option will have an empirical probability higher than that of the preceding subject or option and lower than that of the following subject or option. Thus, the first subject or option will have the lowest (minimum) empirical probability in the entire sample, and the last subject or option will have the highest (maximum) empirical probability in the entire sample, ordering the sample's n th ranking positions in precedence from lowest to highest by empirical probability.

18.1.3.4.3. Given any sample—at least in the initial descriptive, pre-experimental study or initial evaluation (first measurement)—whether of growth or decrease, and with N known, merely being told the ranking position (" n^o ") that a subject or option occupies gives us enough information to know whether that " n^o " lies toward the beginning, the middle, or the end of the sample.

18.1.3.4.3.1. In growth studies, at least in the first measurement, with N known, simply knowing the n th ranking position (" n^o ") tells us whether that subject or option is at the beginning of the ranking (therefore has a low empirical probability), in the middle (intermediate), or toward the end of the ranking (a subject or option with a high empirical probability).

18.1.3.4.3.2. In decrease studies, at least in the first measurement, with N known, merely knowing a subject's or option's n th ranking position (" n^o ") tells us whether, if it is at the beginning of the ranking, it is the subject or option with the highest empirical probability; if it is toward the middle, it is in an intermediate position; and if it is toward the end, then that subject or option is among those with the lowest empirical probability.

18.2. Toward an individual descriptive study according to changes in each subject's or option's position in the ranking

18.2.1. The n^{th} position that a subject or option occupies in the ranking (" n° "), if ordered by precedence—ascending in growth, descending in decrease—is information that tells us the subject's or option's position in the ranking. With N known, at least in the first measurement, by knowing a subject's or option's n^{th} ranking position (" n° "), one can infer whether the empirical probability tends to be high, low, or intermediate.

18.2.2. If the value " n° " (n^{th} position in the ranking) is sufficiently informative, then simply by operating with these values one can obtain individual estimates of the subject's or option's behavior within the ranking—ascending or descending, growth or decrease—given N is known at least in the first measurement.

18.2.3. Since a new ranking must be made for each measurement, no subject or option in the sample is free to keep its place in the ranking unless conditions allow it. No subject or option keeps its " n° " value if it experiences direct or indirect changes.

18.2.4. Indirect and direct change in empirical probability

18.2.4.1. One property of empirical probabilities is that the slightest, seemingly insignificant variation in any subject or option—being a variation in direct score or frequency—changes the sum of direct scores or frequencies. As soon as the sum of direct scores or frequencies changes, all empirical probabilities for all subjects or options change. Even if, by chance, in a future measurement a subject or option keeps the same direct score or frequency obtained in the first measurement, insofar as there has been a change in the sum of direct scores or frequencies—because any other subject or option has experienced a change in its direct score or frequency—then necessarily, even if a given subject's or option's direct score does not change, if any other subject or option changes, the sum of direct scores or frequencies changes. When the sum changes, the quotient of that subject's unchanged direct score or frequency over a sum that has changed will yield a *different empirical probability*. The same subject or option, having the same direct score or frequency in two different measurements, will experience *indirect* changes in its empirical probability if the sum of scores or frequencies changes. This is known as an *indirect change* in the individual behavior of empirical probability.

18.2.4.2. There is said to be a *direct change* in the behavior of empirical probability when the change in empirical probability is attributable to changes in the subject's or option's own direct score or frequency.

18.2.4.3. Typically, in real situations where all subjects or options change at the same time—so long as that simultaneous change does *not* preserve the differential proportions of the first measurement—the results of future measurements, being the product of simultaneous changes but with differential proportions different from the first measurement, will make the empirical probabilities the synthesis of *both* direct and indirect changes.

18.2.4.4. If it happens that all subjects or options change at the same time in a future measurement *while preserving* the differential proportions of the first measurement, then despite direct changes in direct scores or frequencies for all subjects or options, they will still keep the same empirical probability they obtained in the first measurement.

18.2.5. Any change—direct or indirect—in empirical probability will be capable of altering each subject's or option's order of precedence in the ranking, provided that change, direct or indirect, is sufficient for the subject or option to have a different empirical probability that requires a different position in the ranking. Any change in a subject's or option's ranking necessarily implies, by the effect of displacement, changing its initial ranking position from the first measurement to the new position in the new measurement. Any change of position in the ranking implies that a given subject or option vacates a position to move to a new one, which in turn displaces the subject or option that occupied that position in the first measurement. Thus, when a change occurs in the ranking, and the change produces a displacement, if the displaced subject or option takes the position previously held (in the first measurement) by the subject or option that displaced it, then the displacement is due to a *swap of positions*. For example, if a given subject or option displaces the immediately preceding or following subject or option, moving that subject or option into the position the displacing subject or option previously held in the first measurement, this kind of change in the ranking—displacement in the form of a position swap—should not generate any kind of domino effect on the other subjects or options. If this is

not the case, and the displacement is *not* a simple position swap, then the displaced subject must in turn displace a different subject or option, which in turn must displace yet another, and so on. This kind of chain displacement can produce a *domino effect*, whereby a direct change of position in the ranking triggers a chain of *indirect* changes of positions in the ranking.

18.2.6. A change in the *n*th position in the ranking (“*n*°”) for empirical probabilities can result from various factors.

18.2.6.1. Direct changes in the ranking. A subject or option may require a change of position in the ranking due to direct or indirect changes in its empirical probability. That is, a direct or indirect change in empirical probability can cause a direct change in the ranking.

18.2.6.2. Indirect changes in the ranking. A subject or option may experience a change in its *n*th position because it has been displaced, in the second measurement, by changes produced by some other subject or option. That is, a subject or option may occupy an *n*th position in the ranking initially in a first measurement and lose that position as it is occupied in subsequent measurements by a different subject or option which—through direct changes in its own empirical probability—is able to generate a change of positions in the ranking. If the change of positions in the ranking is a simple swap between two subjects or options, there need not be a domino effect or chain reaction; but if not, it can trigger a chained domino effect, producing successive indirect changes in the ranking unless, at some link in the chain, a simple swap of positions occurs.

18.2.6.3. A change in empirical probability can be direct or indirect. *Direct* when the change in a subject’s or option’s empirical probability is the result of changes in that same subject’s or option’s direct score or frequency. *Indirect* when a subject or option keeps its direct score or frequency intact, but changes in the sum of direct scores or frequencies—produced directly by other different subjects or options—end up producing indirect changes in the other empirical probabilities, even though those particular subjects or options experiencing the change indirectly kept their direct scores or frequencies unchanged relative to prior measurements.

18.2.6.4. A change in the ranking can likewise be direct or indirect. A *direct change* in the ranking occurs when a subject or option that undergoes direct changes in empirical probability requires a new position in the ranking, or when, due to indirect changes in empirical probability, it shifts position in the ranking by increasing or decreasing the number of nth positions, depending on the extent of shared nth positions. An *indirect change* in the ranking always entails an indirect change in empirical probability and is an indirect change in the ranking by displacement of the nth position caused by direct changes in the empirical probability of other, different subjects or options.

18.3. Shared positions

18.3.1. It may happen that, within a sample, a set of subjects or options have the same empirical probability. In the order of precedence in the ranking, all subjects or options with the same empirical probability must be assigned the same position in the ranking (“n^o”): that is, what will be known as a *shared position* in the ranking (“n^o”), in this case shared by as many subjects or options as have the same empirical probability.

18.3.1.1. Thus, a position in the ranking (“n^o”) can be held by a single subject or option, provided that the empirical probability for that position is held by that subject or option alone.

18.3.1.2. If the empirical probability associated with that ranking position (“n^o”) is an empirical probability that different subjects or options share at the same time, then the ranking position associated with that empirical probability will be a *shared position* in the ranking for all subjects or options that have that same empirical probability.

18.3.2. A position in the ranking (“n^o”) can, therefore, be occupied by a single subject or option or be shared at the same time by different subjects or options.

18.3.3. As N increases, the usual tendency is for empirical probabilities to approach zero; *human limitation is the source of error*. The only possibility, in a trend toward zero, is *decimal selection*: a decimal selection from possible infinite decimal sequences where it may occur that two or more different subjects or options have identical empirical probabilities due to the error introduced by decimal selection (accumulated error). Consequently, both (or more) subjects or options will, because of the

accumulated error, have the same *shared* position in the ranking (“n^o”). In a possible order of precedence in a ranking of empirical probabilities where there exist empirical probabilities identical at least up to the selected decimal part, it is evident that those identical probabilities must be assigned the same position in the ranking—that is, a *shared* position in the ranking. There is always the possibility that two empirical probabilities apparently identical at least in the selected decimal part might, if longer (but discarded) decimal sequences—or even infinite ones—were considered, differ at some (possibly infinitely far) decimal place. Since this extreme (infinite) detail is unknown, the only thing to do, among two or more empirical probabilities with sufficiently long—or possibly infinite—decimal sequences but identical decimal selection, is to assign them a *shared* position in the ranking.

18.3.4. In a decrease study where the ideal trend is a sample with zero non-ideal qualities—a *sample of zeros*, with all subjects or options having empirical probability zero, the *Minimum Possible Empirical Probability*—evidently, since the last nth position of the ranking would correspond to probability zero, the last position in the ranking would be shared by all subjects or options that have probability zero.

18.3.5. In a positive-bias study where there is only one ideal, insofar as the ideal subject or option tends toward the *Maximum Possible Empirical Probability*, the other subjects or options will simultaneously tend to share the same nth position: the shared nth position in the ranking assigned to the *Minimum Possible Empirical Probability* (probability zero). In an omega study, where the ideal trend is that the ideal subjects or options tend toward the *inversion of omega*, $1/\Omega$, to the detriment of the non-ideal ones whose value is to be reduced to zero, it follows logically that, in an omega study, the subjects or options tending toward the inversion of omega may even reach very similar—or identical—values to the inversion of omega. The nth position corresponding to the inversion of omega will tend to be shared in the ranking by all ideals, while the non-ideal subjects or options will tend to share the nth position of probability zero.

18.3.6. If in studies of equality of opportunity the ideal is that all subjects or options tend to equal the *inversion of N*, then, the more subjects or options have empirical probability equal to the inversion of N, the more the intermediate nth position in the ranking—corresponding to those positions identical to the inversion of N—will be a

position *shared* by all subjects or options whose empirical probability equals the inversion of N.

18.3.7. Summary. Most commonly, in a first measurement, the empirical probabilities of subjects or options will not necessarily tend to be identical to one another. However, in future measurements—second or nth—it is unsurprising that groups of subjects or options may appear that have the same empirical probability at the same time. Whether because, in positive-bias studies, as the ideal tends toward the *Maximum Possible Empirical Probability*, the minima tend equally toward empirical probability zero; or because, in an omega study, the aim is that a set of ideal subjects or options tend to an ideal probability (inversion of omega, $1/\Omega$) to the detriment of the minima (which become lower), it is logical that the minima share probability zero and the omega values tend to be similar to the inversion of omega—those reaching zero sharing the nth position “n^o” for probability zero, and those reaching the ideal sharing the nth position for the ideal probability. In studies of equality of opportunity, if the ideal is that everything tends toward the ideal of equality of opportunity (inversion of N), then in future measurements it is normal for subjects or options to reach empirical probability identical to the inversion of N, sharing the nth position “n^o” assigned to that probability. Conclusion: while in initial measurements—first measurement, initial evaluation, or pre-experimental—there is no reason for subjects or options to have the same empirical probability (beyond chance), in future measurements—second or nth—it is normal for subjects or options to have the same empirical probability, especially in experimental models or those involving a particular direct intervention on reality. All subjects or options that have the same empirical probability will be assigned the same nth position in the ranking; that nth position in the ranking, associated with an empirical probability shared by different subjects or options, will be known as a *shared ranking position*, shared by all subjects or options that have the same empirical probability.

18.3.8. Since there may be subjects or options with the same empirical probability, and all identical empirical probabilities will be assigned a shared position in the ranking, as nth measurements are made it may occur that the number of positions in the ranking decreases.

18.3.9. If, in a decrease study toward zero non-ideal qualities across the entire sample, the ideal is for all subjects or options to tend to probability zero—the *sample of zeros*—and probability zero is shared by multiple subjects or options (sharing the *n*th position in the ranking assigned to probability zero), then the number of positions in the ranking will decrease as the number of subjects or options sharing the *n*th position assigned to probability zero increases.

18.3.10. If, in a study of equality of opportunity, the norm is that all subjects or options have empirical probability identical to the inversion of *N*, then the *n*th position in the ranking assigned to empirical probability equal to the inversion of *N* will be a *shared* *n*th position in the ranking for all subjects or options with empirical probability equal to the inversion of *N*. The more subjects or options share the *n*th position assigned to empirical probability equal to the inversion of *N*, the fewer total *n*th positions there will be in the ranking.

18.3.11. If, in a growth study of a set of subjects or options, the ideal is that the ideal subjects or options tend toward maximum values of the sample distribution, it is not surprising that—if there is more than one ideal subject or option—those values that tend to be maximal might even become identical to the inversion of *omega*, such that those identical values share the same position in the ranking. In any case, even if there were only a single ideal subject or option in the entire sample for increasing positive bias, insofar as that single subject or option would be the only ideal in the whole sample, this implies that there would be *N* minus one non-ideal subjects or options whose ideal, as non-ideal, would be empirical probability zero. If, in a positive-bias study, there are *N* minus one non-ideal subjects or options, then as that single ideal subject or option tends toward the *Maximum Possible Empirical Probability* (one), the rest—the *N* minus one non-ideal subjects or options—will tend toward zero, all of them sharing the same shared position (“*n*⁰”) assigned to probability zero for the negative qualities of the non-ideal subjects or options. If a sample has more than one ideal subject or option, those ideal subjects or options may reach the same maximum empirical probability, sharing the *n*th (“*n*⁰”) position of the maximum, while the rest of the non-ideal subjects or options likewise reach probability zero, sharing the *n*th (“*n*⁰”) position of the *Minimum Possible Empirical Probability*.

18.3.12. Logically, if the more measurements are made the greater the likelihood that more subjects or options share positions in the ranking, then the more measurements are made, the more the number of distinct positions in the ranking will decrease. As the number of subjects or options sharing positions in the ranking increases, **the total number of positions forming the ranking will decrease** proportionally.

18.3.13. The total number of positions in the ranking will be denoted “ N^0 ”.

N^0 = total number of positions in the ranking

18.3.14. As the number of measurements increases—especially in experimental studies or those with direct intervention in reality [Quasi experimental studies can be included in this category, I wrote a post about this type of experimentation by myself in the blog]—it is usual for the total number of positions in the ranking to tend to be smaller than the sample’s N .

$N > N^0/me$

N = number of subjects or options (in infinite universes or universes of limited options)

N^0/me = total number of ranking positions in the n th measurement

18.3.15. If, as more measurements are made, the number of ranking positions normally decreases, then although in the first measurement—knowing N —the n th position “ n^0 ” is sufficient to indicate a subject’s or option’s place in the ranking, in future measurements, since “ N^0 ” (the total number of positions in the ranking) is less than N in experimental or direct-intervention studies, the value “ n^0 ” ceases to be sufficient information in second or n th measurements—in short, in future measurements.

18.3.16. Whatever the reason, there are situations in which more than one subject or option can have the same empirical probability. In such cases, those subjects or options are said to *share the same position* in the ranking; that is, the same “ n^o ” must be shared by all subjects or options with the same empirical probability.

18.4. Homogenization of the ranking of positions

18.4.1. In ranking studies, a sample is said to tend toward homogenization when, in n th measurements, the value N^o tends to be significantly lower than magnitude N , as shared positions increase—so subjects or options in the ranking tend to become homogeneous at certain positions.

18.4.2. The sample may tend to homogenize positions for different reasons: for example, homogenization by subjects or options sharing the n th ranking position corresponding to probability zero (in decrease studies); homogenization toward inversion of N (in equality of opportunity); or, in positive-bias studies, some ideal subjects or options sharing the ideal probability while non-ideal subjects or options simultaneously share the n th position at probability zero.

18.4.3. In any case, any homogeneous trend of the sample is a trend toward increasing shared n th positions among different subjects or options with the same empirical probability, or a trend for subjects or options to have the same empirical probability. For example, if the study’s aim is to reduce all empirical probabilities to zero, then the trend for all empirical probabilities to be zero is a trend to homogenize the sample at probability zero—whether because the ideal is a *sample of zeros*, or because in a growth study there are N minus one non-ideal subjects or options so that those N minus one share the same shared position. Another clear example: in equality of opportunity, the trend to homogenize the sample at inversion of N ; or in omega studies, the trend to homogenize ideal subjects or options at inversion of omega, while non-ideals are homogenized at the Minimum Possible Empirical Probability.

18.4.4. If the object of study is the trend to homogenize the ranking, the ranking study offers another avenue for rational critique of the sample’s homogenizing trend.

18.4.5. A shared *n*th position in the ranking is that *n*th position assigned to an empirical probability simultaneously held by different subjects or options. Most commonly—especially in experimental or real-intervention studies—future measurements (second or *n*th), depending on the experimental variable’s behavior, tend to exhibit more shared *n*th positions.

18.4.6. If, in a decrease study, for example, *all* empirical probabilities equal zero, the ranking would consist of a single shared *n*th position—the number-one position—assigned to probability zero. So long as all empirical probabilities in the sample are zero, that first and only *n*th position in the ranking would be shared by all subjects or options in the sample, making it the only position in the ranking in the case where all subjects or options have empirical probability equal to zero non-ideal qualities—the *sample of zeros*. If, in an equality-of-opportunity study, all empirical probabilities equal the ideal of inversion of *N*, then the ranking would likewise consist of a first and only shared *n*th position—the number-one position—assigned to the theoretical probability equal to inversion of *N*, shared by all subjects or options in the sample (provided they all truly have empirical probability equal to inversion of *N*). If, in a positive-bias study, the ideal is that only one subject or option has the Maximum Possible Empirical Probability (probability one), then the rest will have probability zero; in positive bias there would be only **two** positions in the ranking: the shared position for all subjects or options with probability zero (non-ideal qualities) and the single, non-shared position for the unique ideal subject or option whose empirical probability is non-zero. In omega studies, the trend is likewise toward two *n*th positions in the ranking, but *both shared*: the shared number-one position at probability zero (ideal for all non-ideal subjects or options) and the shared *n*th position at the ideal probability (inversion of omega) for all ideal subjects or options.

18.4.7. Since—especially in experimental or real-intervention studies—the number of ranking positions tends to decrease as shared positions become more homogeneous, the ratio of the total number of *n*th positions in the ranking, N^0 , to the sample *N* of subjects or options, will tend toward zero as N^0 becomes smaller. The *minimum* value of the ratio N^0/N is the inversion of *N*, $1/N$. If all subjects or options share a single *n*th position—because all have the same empirical probability (e.g., all have probability

zero in decrease, or all have empirical probability equal to inversion of N in equality of opportunity)—then the ratio N^0/N equals inversion of N ($1/N$). This yields a new function to be added to inversion of N in the ranking study of empirical probabilities.

18.4.8. Since the *minimum* possible ranking is the one in which all N subjects or options share a single n th position—the only position in the entire ranking—then the total number of ranking positions, N^0 , equals one; the ratio N^0/N equals $1/N$, the inversion of N . In the ranking statistics, inversion of N thus acquires a new function: it is the *minimum probability of homogeneity error* in the ranking.

18.4.9. If the ratio N^0/N tends to zero as N^0 becomes smaller—and as N^0 becomes smaller homogeneity increases—and if in certain studies the ideal is for N^0/N to be smaller, then in a study of ranking homogeneity (by increasing shared positions) the smaller the value of N^0/N , the better. The smaller N^0/N , the smaller the homogeneity error; the smaller the homogeneity error, the greater the homogeneity of the sample. Homogeneity error essentially measures *heterogeneity*—heterogeneity is the error of homogeneity. Homogeneity without error is homogeneity free of any heterogeneity. Therefore, in the ranking study, inversion of N is the *lowest possible homogeneity error* (i.e., the lowest possible heterogeneity).

18.4.10. As N^0 decreases with more shared n th positions—and as more shared n th positions imply greater ranking homogeneity—the ideal is for N^0 to be as small as possible, because the smaller N^0 is, the smaller the sample's homogeneity error. If a smaller N^0 implies a smaller homogeneity error, then the ratio N^0/N should *measure the probability of homogeneity error of the ranking*, i.e., the ranking's heterogeneity. $N^0 : N = \text{probability of heterogeneity (or homogeneity error) in the ranking}$

18.4.11. If the probability of error in ranking homogeneity is the ratio N^0/N , and the minimum number of possible ranking positions is at least one shared n th position for the entire sample—i.e., all empirical probabilities share the same position (e.g., all equal zero in decrease, or all equal inversion of N in equality of opportunity)—then the minimal possible probability of homogeneity error in the ranking occurs when N^0 equals one:

$(N^0 = 1) : N = 1/N$ = minimal probability of homogeneity error in the ranking (in samples of zeros or equality of opportunity)

18.4.12. Thus, in ranking statistics, inversion of N gains yet another function in addition to its others. In ranking statistics—whether for universes of infinite subjects/options or of limited options—if more shared n th positions arise (subjects or options having the same empirical probability), with N^0 the total number of ranking positions and N the number of subjects or options in the sample, the ratio N^0/N is the *probability of homogeneity error* in the ranking. If the minimum possible ranking is that formed by a single n th position shared by the entire sample—because all subjects or options have the same empirical probability (probability zero in decrease, or empirical probability equal to inversion of N in equality of opportunity)—then $N^0 = 1$ and $N^0/N = 1/N$, which is inversion of N . Therefore, in ranking statistics, inversion of N ($1/N$) is the *minimum probability of error in ranking homogeneity* in samples of zeros and equality of opportunity. In positive-bias studies, when there is only a single ideal subject or option (the only one to increase its normal positive bias), the total number of ranking positions, N^0 , will be **two**: the shared n th position for the $N - 1$ non-ideal subjects or options with probability zero, and the n th position for that single ideal subject or option. If a sample has more than one ideal subject or option (omega models), the total number of ranking positions should ideally also be **two** shared n th positions: one shared by all non-ideal subjects or options (probability zero) and the other shared by all ideal subjects or options at the ideal probability (inversion of omega). In such positive-bias and omega models, the ideal would be that the ratio of the total number of ranking positions, N^0 , to the sample size N equals **$2/N$** (i.e., two shared n th positions divided by N).

$(N^0 = 2) : N = 2/N =$ minimum probability of homogeneity error in the ranking in positive-bias studies and omega models.

18.4.13. If the inversion of N , $1/N$, is the minimum probability of error in ranking homogeneity in equality of opportunity or in negative-bias studies tending toward samples of zeros, and the minimum probability of homogeneity error in the ranking in positive bias and omega is two over N , $2/N$; and if the probability of error in the ranking equals the quotient of N^0 (total number of ranking positions) divided by N (number of subjects or options in the sample), then one can compute the **Validity of Ranking Homogeneity**.

18.4.13.1. *Validity of Ranking Homogeneity* in equality of opportunity or negative bias when the trend is toward samples of zeros: equal to the difference of a critical probability minus the difference of the quotient ($N^0 : N$) minus the inversion of N . Zero or positive is accepted as homogeneous. The critical probability equals the product of the error percentage X (out of one hundred) and the inversion of N .

Validity of Ranking Homogeneity in equality or trend toward samples of zeros =

$$p(xc) - [(N^0 : N) - 1/N] = \text{zero or positive is accepted}$$

$$p(xc) = 1/N \cdot (X : 100)$$

X = error percentage

18.5.1.3. However, this seemingly simple idea—that in growth the maximum should occupy the last position in the ranking (ordered ascending from the first position), or in decrease the minimum should occupy the last position (ordered descending)—and therefore the ideal ranking position in growth or decrease is always to occupy the last position in the ranking, implies that if N^0 is constant then every growth or decrease must be verified such that a subject or option, according to its ideal, occupies higher positions in the second measurement; hence only a positive difference (second minus first position) would be accepted. This *a priori* simple idea proves complex in practice because, even if the ideal is an increase in the n th position, in reality this will not be evident: as shared positions increase, magnitude N^0 will *not* be constant— N^0 will tend to shrink. Thus, the ideal may be achieved (a subject or option, in growth or decrease, moving to the last positions), yet as N^0 tends toward homogeneity, the last position that the maximum or minimum occupies can, in fact, be *lower* in the second measurement than it was in the first, due to the increase in shared positions and the resulting reduction of N^0 between two different measurements.

18.5.1.4. It may happen in growth studies that a subject or option has a *lower* n th position in the ranking in the second measurement than in the first, even when its empirical probability in the second measurement is *higher* than in the first. This can easily occur because, if the ranking tends to homogenize into shared positions, then with a smaller total number of ranking positions (N^0), a subject or option may have a higher empirical probability in the second measurement than in the first, but—because the total number of positions N^0 has decreased—its ranking position in the second measurement may nonetheless be lower than the one it held in the first. If shared n th positions have increased, the reduction in the total number of ranking positions means that subjects or options whose empirical probability increased may nevertheless *not* have higher ranking positions than in the first measurement.

18.5.1.5. The same can occur in decrease studies: a subject or option may tend sufficiently toward zero (or be nearly zero) and come to occupy the last ranking position in the second measurement; yet as shared positions increase, its ranking position—despite being the last—may, because N^0 in the second measurement is lower than N^0 in the first, yield a *negative* difference (second minus first position).

18.5.1.6. Therefore, the differentials of ranking positions between two different measurements must incorporate the homogenizing trend of the sample.

18.5.1.7. The **Ranking Level**, whenever positions are ordered from the first position—ascending in growth, descending in decrease—and with the ideal in all cases being: in growth, that the maximum occupies the last ranking category; in decrease, that the minimum occupies the last ranking category; then the Ranking Level (for growth or decrease) equals: the product of the ranking position in the second measurement and the quotient of total number of ranking positions in the first measurement, divided by the total number in the second measurement, and the result minus the ranking position in the first measurement. A positive result indicates growth or decrease (according to the study's aim).

Ranking Level (ascending in growth or descending in decrease) =

$[n^o/2^a \cdot (N^o/1^a : N^o/2^a)] - n^o/1^a = \text{positive is accepted}$

$n^o/2^a$ = nth position in the second measurement

$n^o/1^a$ = nth position in the first measurement

$N^o/1^a$ = total number of ranking positions in the first measurement

$N^o/2^a$ = total number of ranking positions in the second measurement

18.5.2. Ranking Significance

18.5.2.1. The Ranking Level accepts any variation that is verified as positive between the positions a given subject or option may have in two different measurements: multiply the second-measurement ranking position by the quotient of total number of ranking positions in the first measurement over those in the second, and subtract the ranking position the subject or option held in the first measurement.

18.5.2.2. However, the variables introduced in Ranking Level may not be rationally sufficient. It is not enough to introduce subjective, sensory variables; one must introduce objective, rational variables. It is not enough to validate a change in behavior sensorially; one must validate whether that sensed change is also a sufficiently rational variation. The fact that reality undergoes changes does not imply that the perceived (*physiological*, sensory) change is sufficient to demonstrate a *trend*; a change by itself is not a rational trend—it is a mere sensory appearance. To demonstrate that a change is truly a rational trend, a rational variable must be introduced. Thus, over the purely subjective model—changes in position between two measurements, plus a N^0 -magnitude correction for the homogenizing effect of shared positions—we introduce a rational variable (the true objective essence), a *critical reason*, equal to the product of the reliability percentage X (out of one hundred) and the difference of (N^0 in the first measurement minus the ranking position in the first). The difference between the Ranking Level and this rational variable will be called **Ranking Significance**, valid for growth studies (ordered ascending from the first position) or for decrease studies (ordered descending from the first position).

18.5.2.3. Ranking Significance equals: the difference of { (second-measurement ranking position) times (N^0 in the first measurement divided by N^0 in the second) } minus (first-measurement ranking position), and then minus the critical value “nc”. Zero or positive is accepted. The critical value **nc** equals the reliability percentage X (out of one hundred) times the difference of N^0 in the first measurement minus the first-measurement ranking position. For both growth and decrease studies in the described ranking order.

Ranking Significance (ascending in growth or descending in decrease) =

{ [$n^0/2^a \cdot (N^0/1^a : N^0/2^a)$] – $n^0/1^a$ } – nc = zero or positive is accepted

nc = ($N^0/1^a - n^0/1^a$) · (X : 100)

X = reliability percentage

18.5.3. The importance of ordering in precedence from least to greatest in growth, and from greatest to least in decrease

18.5.3.1. For the equations just presented to be valid, the order of precedence must be done as described at the start of this section: order in precedence from least to greatest in growth (ascending), and in decrease studies order from greatest to least (descending).

18.5.3.2. This way of ordering in precedence—least to greatest in growth (ascending), or greatest to least in decrease (descending)—is a way of organizing real measurements that will have important applications in the *Impacto del Defecto*, where defect types by severity are ordered from lesser to greater severity, and for the study of *Distribución Efectiva*, where each ideal category of effectiveness will be ordered in precedence from

lesser to greater categorical importance of effectiveness within a process or system [in fact, Ranking analysis is a development of Impact of the Defect and Effective Distribution, both of them with some relevance in the GAI-GPS system, but it is obvious, to build such machine, we need more mathematics, all possible mathematical analysis systems available, not only Second Method, and we need more research, and of course, we need more time, especially regarding the integration of cyborg robotics within the GAI-GPS system].

18.6. Alternative forms of order of precedence in the ranking

18.6.1. The forms of order of precedence in the ranking explained above—ascending in growth or descending in decrease—are logical ordering forms consistent with their graphical expression. Graphically, any ascending trend is quickly identified with growth, and any descending trend is quickly identified with decrease .

18.6.2. Ordering the n th positions descending in decrease (greater to lesser), or ascending in growth (lesser to greater), is one possible form among other possible forms. If the positions were ordered in a bar chart on Cartesian axes, the ideal—so as to convey growth or decrease—would be to order ascending from least to greatest when studying growth, or descending from greatest to least when studying decrease, always as possible forms among the possible orders of precedence for the ranking.

18.6.3. Other alternative ways of ordering the ranking can exist.

18.6.4. In fact, the most common practice—for example, at the Olympics when athletes are ranked—is that the number one in the ranking is the best athlete, and the ranking follows an order of precedence from the best athlete (who occupies the first n th position in the Olympic ranking) to the rest. At the Olympics, athletes may be ranked by who has the highest number of points—ordering from the first position with the most points down to fewer points—or by the smallest stopwatch time each athlete achieves in an event—ordering from the first position (least time) to the greatest time required.

18.6.5. Similarly, analogous orders of precedence could be used in statistical ranking studies.

18.6.6. An alternative way of ranking in a growth study, for example, would be to order from greatest to least. If, in growth, the ideal is to have the highest empirical probability, order the empirical probabilities for the *n*th positions in descending order so that the number-one position in the ranking is for the maximum empirical probability, and the last position is for the minimum empirical probability. Thus, there is the alternative of a descending order of precedence in the ranking from the first position, greatest to least.

18.6.7. An alternative way of ranking in a decrease study, for example, is to order from least to greatest. Thus, if the ideal in decrease is the smallest possible empirical probability, the minimum empirical probability will have the number-one ranking position, and the maximum empirical probability the last ranking position, always ordering from the first *n*th position in the ranking from least to greatest.

18.6.8. *The biggest problem with this alternative way of ordering—greatest to least in growth from the first position, or least to greatest in decrease from the first position—is that, if a graphical representation of the ranking were made on Cartesian axes, one would not clearly perceive decrease as descending or growth as ascending.*

18.6.9. If, in a growth study, one ranked as in the Olympics by points—for example, first position to the athlete with the greatest number of points (the maximum), so that each prior athlete has more points than the next, ordering greatest to least so the first position is occupied by the maximum and the last by the minimum—or, in the Olympics, in a decrease study of athlete stopwatch time, ordering athletes by their time so that the athlete with the smallest time (the minimum) has the first ranking position and the others follow with larger times (the maximum then being last)—if the ranking were ordered as just described (in growth, first position to the maximum; in decrease, first position to the minimum), then the **Ranking Level** would take the following form:

Ranking Level when the maximum in growth or the minimum in decrease is the first position =

$$n^0/1^a - [n^0/2^a \cdot (N^0/1^a : N^0/2^a)] = \text{positive is accepted}$$

18.6.10. Then, if the ranking is ordered so that in growth the maximum is the first position, or in decrease the minimum is the first position, the **Ranking Significance** would be:

Ranking Significance when the maximum in growth or the minimum in decrease is the first position =

$$\{ n^0/1^a - [n^0/2^a \cdot (N^0/1^a : N^0/2^a)] \} - nc = \text{zero or positive is accepted}$$

$$nc = n^0/1^a \cdot (X : 100)$$

X = reliability percentage

18.6.11. The main advantage of ordering the ranking by precedence according to the model described initially in this section—ranking ascending in growth from least to greatest starting at the first position, and ranking descending in decrease from greatest to least starting at the first position—is that, if the ranking is plotted

on Cartesian axes, it will be easier to perceive growth as an ascending order of the ranking, and decrease as a descending order of the ranking.

18.6.12. The fundamental reason for studying the ranking in ascending order for growth and descending order for decrease is to have a more graphical sense of the trend; an ascending graphical representation of growth, or a descending one of decrease, is a more realistic representation, and on that basis the levels or Significance of Growth or Decrease have been ordered.

18.7. The condition of possibility for the ranking study

18.7.1. The ranking study just described is not equally valid for every kind of study; there will be studies where it fits better and others where it will hardly be advisable to apply it.

18.7.2. The most important condition of possibility for the ranking study is that the sample variable N (number of subjects or options) be sufficient.

18.7.3. Not all samples are suitable for ranking studies. For example, a sample made up of only two options—the minimal sample for a stochastic study—would not be a recommendable model for a ranking study; the very idea of ranking usually implies a much broader sample concept than one limited to just two options. A different matter is that, on a sufficiently large N , in a positive-bias or omega model study, one examines how the sample's shared positions, N^0 , tend to shrink— as shared positions increase— down to a sample of two shared positions: the n th position shared by the non-ideal subjects or options (probability zero) and the second n th position for the ideal subjects or options—either a non-shared second position for the *single* ideal subject or option in positive-bias studies that reaches the Maximum Possible Empirical Probability, or a second *shared* n th position in omega models for all ideal subjects or options. Whether the ideal trend is a sample of two n th positions (positive bias or omega models) or the ideal is a single position shared by the entire sample (samples of zeros and equality of opportunity), ideally the sample of subjects or options, N , should be sufficiently large for the ranking study to be useful.

18.7.4. In principle, one may say that ranking study is universally valid for any study in universes of *infinite* subjects or options; however, ranking study would *not* be advisable for just any study in universes of *limited* options. The more limited the number of options, the less useful the ranking study is. The larger the number of options in a limited-options universe, the more sense the ranking study makes. In limited-options universes, the ranking study should be *discarded* when few options make up the universe, and *adopted* when many options bound the universe.

18.7.5. Scientific policy should decide from what number of subjects or options it is advisable to apply the ranking study model. Ideally, to apply the ranking study, at least in the first measurement the *Sample Sufficiency of N* or the *Critical Requirement of Normality* should be met, with a sufficient number of subjects or options. Especially in limited-options universes, it is important to consider from what number of options it becomes advisable, since the nature of ranking makes it ideal for studies where the sample N is at least considerably broad. Scientific policy should decide the types of study for which this kind of test is suitable.

18.7.6. While in universes of infinite subjects or options the ranking model is applicable to any possible model of empirical study of reality, in limited-options universes scientific policy must decide when the ranking model can be applied.

18.7.7. *A priori* it is difficult to say when a sample's N is sufficiently large to apply the ranking model, because the type of rational critique information will depend on the study's nature.

18.7.8. Depending on the type of universe (limited options, or infinite subjects/options), on the object of study (equality of opportunity or bias, positive or negative), on whether it is a growth or decrease study, whether it is a study of change in behavior by chance or reliably, whether it is a study of reliability or error, and depending on the aims pursued by the research—according to the factors involved in each particular study—scientific policy must select a model of rational critique to test the isomorphism between idea and reality, in order to decide what happens within the margin of contradictions it is willing to accept in the political theory of science.